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EXPERIMENTS ON VISCOPLASTIC RESPONSE OF CIRCULAR PLATES TO IMPU--ETC(U)

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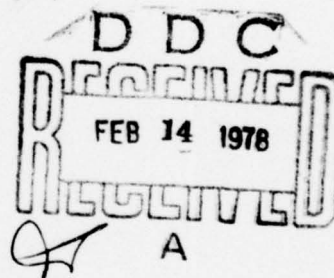
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EXPERIMENTS ON VISCOPLASTIC RESPONSE OF CIRCULAR PLATES  
TO IMPULSIVE LOADING

S. R. Bodner and P. S. Symonds



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"Experiments on Viscoplastic Response of Circular Plates  
to Impulsive Loading"<sup>1</sup>

by

S. R. Bodner<sup>2</sup> and P. S. Symonds<sup>3</sup>

ABSTRACT for	
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Abstract

Tests are described on circular plates of mild steel and commercially pure titanium loaded impulsively by means of explosive sheet. Three loading geometries were used, with magnitudes such that final deflections in the range from one to about seven plate thicknesses were produced. Clamping against radial as well as transverse deflections at the edge was provided. Both materials exhibit strong plastic rate sensitivity. Parameters describing this behavior were obtained from stress-strain tests at low to intermediate rates together with published data for high strain rates. The measured final deflections and response times are compared with predictions of the mode approximation technique as extended to large deflections of viscoplastic structures.

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## 1. Introduction.

The main purpose of the work reported here was to investigate the relation between deflections estimated by the "mode approximation" technique [1 - 4] and the actual deflections of pulse loaded structures of strain rate sensitive plastic material behavior. This estimation technique involves an intrinsic approximation, as well as certain idealizations and simplifications. The method has the advantage, among others, that most of the errors are qualitatively understood, and in most cases are expected to make for over-estimates of deflections. Comparison with experiments are obviously desirable to check on the correctness of these expectations and guide future work. Further discussion of error will be given later. The details of the application of the mode approximation technique to this problem are presented in a companion paper [4].

The structure studied here is a circular plate with full edge clamping. The plate is subjected to a short duration--high intensity pulse of pressure. This is idealized in the estimation technique as impulsive loading (initial velocity imposed "instantaneously" with negligible initial displacements).

Our interest is in the range of loading such that both flexural and extensional response must be considered. Thus the final deflections range from about one plate thickness to approximately seven thicknesses.

Although the clamped circular plate has received considerable attention in research there seem to be remarkably few experimental studies covering this range. Recent work has been mainly concerned with numerical techniques [5, 6] or analysis [7], and has included limited tests and comparisons. An historical summary has been given by Jones [8]. The present experimental investigations were meant to be fairly comprehensive. When only a few tests are made, one can generally obtain some sort of "confirmation" of a proposed theory, but one cannot



truly evaluate a new approach or obtain guidance for improving it. This is especially true for an estimation technique where understanding the effects of deliberate approximations and idealizations is of prime importance. Thus we have used two materials, mild steel and titanium (whose dynamic plastic behaviors differ considerably quantitatively); and have used three loading geometries, since the ability to treat arbitrary specified loading with minimal computational effort is a key feature of the mode approximation technique. For each material and loading distribution the full range of impulse was applied, as outlined above. In each test direct measurement was made of the total impulse on the structure so as to eliminate questions of calibration. The main response measurement in each test was the final mid-point deflection and deflection profile. Approximate measurements were also made of deflection-time history, furnishing information about times of reaching peak or final deflection magnitudes.

## 2. Test Techniques and Apparatus

We chose to test circular plates of 63.5 mm (2.50 in.) diameter and thickness averaging either 1.9 mm (0.076 in.) (steel) or 2.3 mm (0.092 in.) (titanium), and which were clamped in a manner intended to prevent displacements in the plane of the plate as well as transverse displacements and slopes. To provide this "fully fixed" condition we used the clamping frame shown in Fig. 1. The specimen plate formed the central circular area of a square plate 114 mm (4.5 in.) on each side, which was clamped between steel blocks of the same square dimensions and 22 mm (7/8-in.) thickness, by eight 6.4 mm (1/4-in.) bolts (Fig. 1b). It was intended that with sufficient tightening of the bolts, friction forces would prevent in-plane displacements at the edge, when finite transverse displacements were produced by impulsive loading. In fact at the largest load magnitudes these forces did not prevent inward displacements, as will be discussed

later. Attempts to improve the constraint against slipping by inserting a ring shim stock, and by using epoxy adhesive, were unsuccessful.

As indicated in Fig. 1a, transverse pulse loading was applied by detonating a disk of explosive sheet. This was separated from the plate by a styrofoam buffer pad of the same radius and 1/4 in. thickness. This pad served to lengthen the pressure pulse acting on the plate and reduce the pressure magnitude; without it, failure by shear may occur and a hole blown through the plate. In some preliminary experiments we used one or two layers of 3 mm (1/8-in.) thick neoprene, but the much lighter (weight about 0.5 gram or less) styrofoam pads were considered preferable and were used in all tests reported here. They disintegrated completely under the explosion. No evidence was observed of shear deformation at the edge of the loaded area.\* The sheet explosive used was DuPont Detasheet "C" (2.0 mm (0.08 in.) nominal thickness). Variations of loading were obtained by using thinner sheets. These were obtained by one or more passes through a rolling mill, producing thicknesses as small as 0.43 mm (0.017 in.). Larger loads were obtained by using more than one thickness.

A series of tests was made at each of three loading geometries, defined by the ratio  $a/R$  of the loaded area radius to the plate radius, Fig. 1a. We used  $a/R = 1, 1/2, 1/3$ . This program of tests was designed to check the intrinsic approximation of the mode technique, as will be discussed later.

The primary measurements in each test were the initial impulse imparted to the specimen, and the final deflection magnitude. Measurement of the displacement-time history of the central part of the plate was also attempted by means of a device patterned after a condenser microphone; this will be described later. The basic measurement of impulse was made by means of a ballistic pendulum [9]. As sketched in Fig. 2, the specimen, clamping frame, and condenser microphone were

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\* Shear failures were observed in some cases when neoprene pads were used. The mass of these pads also caused some questionable results.

mounted at one end of the suspended I-beam. The impulse applied to the specimen was transmitted to the pendulum through the supporting steel rods. If both the pulse time and the time in which the specimen reaches its maximum deformation are short compared to the natural period of the pendulum, the impulse is related to the swing of the pendulum by

$$I = \frac{W}{g} \frac{2\pi}{T} X_m \quad (1)$$

where  $X_m$  is the total pendulum displacement (at its mass center),  $W$  is the weight of the pendulum, and  $T$  is its natural period of vibration. In the present tests the period was 3.3 sec., while the duration of the plate response was in the neighborhood of 80  $\mu$ sec and the pressure pulse duration was estimated to be less than 10  $\mu$ sec. The effects of motion of structure are negligible in our tests, both durations being less than  $10^{-4}$  times the pendulum period.

It is of course assumed that the impulse measured is due solely to the impulsive pressures loading the plate specimen, with no contribution from explosive gas pressure or from material particles striking the wall of the clamping frame. In the present case such effects are believed negligible. The desirability of making direct impulse measurements in each test is worth emphasizing. The impulse per unit weight of charge depends not only on the charge weight, but on the geometry of the charge and the configuration adjacent to the specimen; it also may depend on the specimen properties [5].

For use in the estimation technique the initial velocity distribution was taken as a uniform velocity  $V_o$  over the nominal loaded area and zero elsewhere. We take  $V_o$  as due entirely to impulsive pressures, so that

$$V_o = \frac{I}{\pi a^2 \rho} \quad (2)$$



where  $\bar{\rho}$  is the mass per unit area of middle surface of the specimen plate and  $a$  is the radius of the explosive sheet. It is evident from Fig. 1a that the high pressure area extends somewhat outside the radius  $a$  of the explosive sheet. This introduces some uncertainty into the comparison between test and theoretical results, but in the mode approximation technique it is easy to take this into account by estimating the changes in deflection due to changes in the loading radius, as will be seen.

The condenser microphone used to record displacement as function of time was similar to ones used by Davies [10] and Kolsky [11], and consisted of a brass plate 10 mm (0.4 in.) diameter held at a fixed distance from the clamping frame. Its initial distance from the rear surface of the plate specimen was 25.4 mm (1 in.) while the maximum transverse plate displacement was about 13 mm (0.5 in.). As indicated in Fig. 2, the condenser microphone was mounted on a cylinder of length such that the time required for elastic waves to travel from the specimen to the condenser plate was about 150  $\mu$ sec, so that effects of these waves could appear after the completion of the specimen's deformation. The electrical circuit is shown in Fig. 3. The insulated condenser plate was held at 450 V. above the grounded specimen plate. The deflection of the plate changes the effective capacitance which produces a change in signal on the oscilloscope. The time constant for the circuit was 0.01 sec which is large compared to the plate response time of about 100  $\mu$ sec, which makes the measured response times fairly reliable. Timing was checked by an independent contact wire system with good correlation. The instrument response is a nonlinear function of the average displacement of a not-well-defined central region of the plate, as seen from the geometry of the arrangement (Fig. 1a). No absolute amplitude calibration was attempted. We used the device mainly to obtain information about the time of reaching maximum or final displacements. A typical record is shown in Fig. 4.



The measured response times and their comparison with predicted times will be discussed subsequently.

It should be mentioned that the condenser microphone system had only about 50 percent reliability. Despite extensive shielding and grounding to isolate the system from ionization effects of the blast and from broadcasting signals, frequent replacement and testing of electrical components, and many other efforts, it remained susceptible to erratic malfunctioning, so that the "displacement" record went off scale immediately. We are inclined to attribute this malfunctioning to effects of the blast pressure or noise, but were unable to eliminate it.

### 3. Material Properties

We tested plates of two strongly rate sensitive metals, namely hot rolled mild steel (ASTM A415) and 99.2 percent pure titanium (Ti-50A). These were both supplied as sheets, and were tested "as received". Tensile tests using coupons cut in the rolling (longitudinal) and transverse directions showed negligible differences in the case of steel, but we observed differences of about 4 - 7 percent between the two directions in the stress-strain curves of the titanium sheet, the stresses in the transverse direction being larger. We averaged results of tests in the two directions to obtain values of material parameters.

The estimation techniques being investigated require specification of the viscoplastic material behavior. This may be described by a family of curves of stress versus strain rate at fixed plastic strain levels, which are usually derived from a family of stress-strain curves from tests run at various strain rates. In our plate tests the maximum strain rates are probably of the order of  $100 \text{ sec}^{-1}$ , and stress-strain data up to at least this rate are required. Tests at high strain rates are expensive, and unfortunately were beyond our budget.

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\* We found strain gages to be unworkable because they spalled from the plate surface.

We therefore made stress-strain tests at four extension rates up to about  $0.1 \text{ sec}^{-1}$  using a conventional machine (Instron). We combined the data so obtained with published data to obtain values for the viscoplastic behavior of our materials. We assumed that results of uniaxial stress tests can be expressed by an equation in the following form

$$\frac{\sigma}{\sigma_0} = 1 + \left( \frac{\dot{\epsilon}}{\dot{\epsilon}_0} \right)^{1/n} \quad (3)$$

where  $\sigma$ ,  $\dot{\epsilon}$  are tensile (or compressive) stress and strain rate, respectively, and  $\sigma_0$ ,  $\dot{\epsilon}_0$ ,  $n$  are material constants of strain rate sensitivity. In general these apply to a specified plastic strain level; however, for mild steel  $\sigma$  is here taken as the lower yield stress. With three available constants, Eq. (3) is capable of fitting strain rate test data for strongly rate dependent metals very closely [2], at least over the strain rate range relevant to structural applications. Our procedure was to obtain  $\dot{\epsilon}_0$  and  $n$  from data in the literature for similar metals [12 - 14], and to determine  $\sigma_0$  for our materials from stress-strain curves at four nominal strain rates, namely  $\dot{\epsilon} = 10^{-4}$ ,  $10^{-3}$ ,  $10^{-2}$ , and  $10^{-1}$ ,  $\text{sec}^{-1}$ . Consistency of the four values so determined provides evidence on the suitability of the whole set of constants for the materials of our tests. In the case of the mild steel, taking  $\dot{\epsilon}_0 = 40 \text{ sec}^{-1}$ ,  $n = 5$ , the values of  $\sigma_0$  agreed very closely with each other, having an r.m.s. variation of only 130 psi from the mean value 32,400 psi. For our titanium the spread was larger, and a systematic trend with the four strain rates was observed, indicating that the values  $\dot{\epsilon}_0 = 120 \text{ sec}^{-1}$ ,  $n = 9$  obtained from published data were less suitable. However, in the worst case the r.m.s. variation from the

mean was only 4 percent of the mean. The values of  $\sigma_0$ ,  $\dot{\epsilon}$ , and  $n$  obtained by this method are regarded as certainly adequate for our purposes.

Figures 5 and 6 show typical tensile test records for our steel and titanium respectively. As strain rate is increased the curves for steel and titanium change in shape as well as in stress magnitude, and the contrast is interesting: for the transversely cut titanium specimens the "upper yield" phenomenon disappears, while for steel it is accentuated.

In Table 1 are listed numerical values of parameters for our tests.

Table 1

		<u>Steel</u>	<u>Titanium</u>
Strain Rate Constant <sup>1</sup>	$\sigma_0$ psi	32,400 <sup>2</sup>	36,400 at $\epsilon^P = 1\%$ (35,200 long., 37,700 trans.)
			38,500 at $\epsilon^P = 2\%$ (37,800 long., 39,200 trans.)
Strain Rate Constant <sup>1</sup>	$\dot{\epsilon}_0$ sec <sup>-1</sup>	40	120
Strain Rate Constant <sup>1</sup>	$n$	5	9
Mass Density	$\rho$ lb sec <sup>2</sup> in. <sup>-4</sup>	$0.73 \times 10^{-3}$	$0.42 \times 10^{-3}$
Plate Radius	$R$ in.	1.25	1.25
Plate Thickness (Average)	$H$ in.	0.076	0.092

<sup>1</sup>As used in Eq. (3).

<sup>2</sup>Lower yield stress.

#### 4. Test Results

Series of tests were made for both mild steel and titanium at each of three values of impulsive loading radius  $a$ , such that  $a/R = 1/3, 1/2$ , and  $1$ . In each series impulses were applied of magnitudes chosen to give final midpoint deflections in the range between about one and seven thicknesses. The main results are shown in Figs. 7-17. With the test results are given estimated curves according to the extended mode technique, and in most cases also according to the simplest mode theory for small deflections.

Comparisons between predicted and measured quantities will be commented on in the final section, particularly with regard to the errors anticipated in the estimation technique. The main experimental errors, i.e., places where the intended experimental conditions were not achieved, were probably in the clamping and in the geometry of loading. As already mentioned, the clamping frame used was unable to prevent slippage at the largest load magnitudes. Pull-in, i.e., slippage, was monitored by the movement of a carefully scribed fine line on the specimen plate immediately adjacent to the clamping frame. It seems that for conventionally clamped plates subjected to large impulsive loading, localized plastic thinning and stretching occurs next to the edge which cannot be prevented by better bonding or higher clamping forces. The largest loads were those with  $a/R = 1$ . For these tests the inward displacement at the edge of the plate was generally less than 1.6 mm (1/16 in.), but was significant with regard to the plate response. The transverse deflection is sensitive to this defect in the clamping condition, and for these cases the measured deflection was considerably larger than it would have been with full clamping. No theoretical correction was attempted for this condition; in estimating deflections, full edge fixity was assumed.

The other experimental error, evident from the loading geometry, is the spreading of impulsive pressure outside the nominal radius  $a$  of the explosive



sheet. An accounting for this in the theory was made by showing estimated deflections for  $a/R = 0.55$  as well as for 0.50; and for  $a/R = 0.40$  as well as 0.33; this required a trivial further calculation.

The deflection and time estimates were derived taking  $\sigma_0$  for steel as the lower yield stress, and for titanium taking  $\sigma_0$  in the first instance as the value corresponding to plastic strain  $\epsilon^p = 0.01$ . Since the titanium exhibits relatively high strain hardening, an additional curve is shown for titanium, in each comparison, namely one computed using  $\sigma_0$  corresponding to  $\epsilon^p = 0.02$ . Thus the spread is shown due to changes in both  $a$  and  $\epsilon^p$ . All numerical values used are shown in Table 1.

Figures 7 and 8 show typical final deflection shapes for steel plates; Fig. 9 shows one for a titanium plate. For the more concentrated loadings (Figs. 7, 9) the measured deflections are smaller than those estimated by the mode technique, but the result in Fig. 8 for loading over the whole plate shows test deflections somewhat larger than the estimated ones. The effect of pull-in is illustrated here.

The final deflections in the six series of tests - three loading radius values for both materials - are summarized in Figs. 10-15 (fifty-seven tests in all). The ratio of the final deflection to specimen plate thickness as function of impulse are shown in Figs. 10-12 for steel plates and for titanium plates in Figs. 13-15. The principal test data are summarized in Table 2.

The edge constraint against radial motion is fairly well satisfied in the tests for  $a/R < 1$ , as in Figs. 10, 11, 13, 14; little or no inward displacement was visible in these tests. For the larger loads used with  $a/R = 1$ , as in Figs. 12 and 15, the pull-in was found in some cases to be as much as 1.6 mm (1/16 in.). In these cases the measured deflections lie above or only slightly below the curves shown for the estimation technique. In the other cases the

observed final deflections lie below the estimated curve. It is seen that the extended mode technique predicts deflections much closer to the observed ones than does the mode method for small deflections, where only bending, without membrane action, is considered; the curves for "small deflections" shown in Figs. 10-15 refer to this simplest form of the mode technique.

In Figs. 13 and 15 for titanium plates, test points are shown marked as having "weaker axial constraint." In one of these cases the plate was thinner than the average, while in the other the area of clamping was smaller. Still larger deflections are observed in both these cases, the importance of radial constraint being further emphasized.

Some illustrations of measurements of the duration of the response are shown in Figs. 16 - 21, the time in microseconds being plotted as function of impulse. Two time values are generally obtained from each test, marked  $t_f'$  and  $t_f''$  in Fig. 4. The typical time history shown in Fig. 4 shows elastic vibrations and a later downward trend, probably associated with the time constant of the circuit (Fig. 3). The time  $t_f''$  is obtained as the intercept of a line approximating the late-time response with the signal trace before reaching the maximum;  $t_f'$  is the time to reach the peak displacement. In view of the nonlinear relation of the signal to the displacement, these times do not have precise significance. Neither corresponds to the response time obtained in the mode technique, in which the deflection increases monotonically to its maximum and final magnitude. The estimated response time first increases with the impulse, but beyond a certain impulse exhibits a slow decrease. The observed response times show practically no variation with impulse, although in Figs. 18 and 21 a slight decreasing trend is noticeable. For steel plates with  $a/R = 1$ , both times are longer than the estimated duration time. This result is undoubtedly related to

pull-in at the edge, which was most pronounced here. It is seen that the observed response times vary with impulse in a manner much more like that predicted by the extended mode technique, than that obtained from the simple small deflection theory. As shown, for small deflections the response time steadily increases with impulse, in an almost linear manner. The observed rise to a maximum followed by a slow decrease can be understood in terms of the transition from flexural to membrane action at finite deflections. In a linear elastic string or membrane the time to reach peak deflection would of course be independent of the starting conditions. At large plastic deflections a beam or plate approaches behavior resembling that of a taut string or membrane, respectively. With the additional inclusion of strain rate dependence, one would expect that the structure would be effectively "stiffer" for larger peak deflections; this would cause the time to reach maximum amplitude to decrease with larger impulse values.

## 5. Discussion

The elements of the extended mode approximation technique are outlined in the Appendix. Here we comment briefly on the test results. Our aim was to check the application of the extended mode technique to a structure which is sensitive to finite displacement magnitudes, and which becomes "stiffer" at finite deflections due to the change of the stress field from flexural to membrane action. We wanted to study the "intrinsic" approximation involved in the mode technique (in the original and extended forms), and to assess its relative importance compared to other approximations and idealizations.

The initial amplitude of the mode form velocity field is found by Eq. A5 (see Appendix). In the present tests the specified initial velocity is  $V_0$  inside radius  $a$ , zero elsewhere. The "matching" by use of Eq. A5 is illustrated by the inset sketches in Figs. 7-9, where  $\dot{w}_0$  and  $\dot{w}_h^0$  are non-dimensional

velocity magnitudes of the given distribution and the mode field, respectively. Typical ratios  $\dot{w}_H^0/\dot{w}_0$  are shown with the sketches. When the initial velocity is sufficiently concentrated, as in Fig. 7, the ratio  $\dot{w}_H^0/\dot{w}_0$  is less than one; this implies that the final midpoint displacement by the mode method will be too small. Conversely when the given velocity distribution is sufficiently spread out, as in Fig. 8, the mode method will give an over-estimate. Since the actual and mode solutions converge, the error in displacement is usually much less than that in the initial velocities. This "intrinsic" error of the mode approximation technique is evidently negative, nearly zero, or positive according as  $a/R$  is  $1/3$ ,  $1/2$ , or  $1$ . Thus in Figs. 10-12, if no other sources of error were present, the predicted displacements from the mode technique should, respectively, lie below, close to, and above the actual test deflections. No such trend can be seen in these figures (for steel) or in those for titanium plates, Figs. 13-15. For  $a/R = 1/3$  and  $1/2$ , the test deflections lie below the mode estimates, while for  $a/R = 1$  the test deflections are either above (steel) or slightly below (titanium) the predicted curves.

These results can be explained in terms of further idealizations and approximations in our application of the mode technique, as summarized below:

- (1) A homogeneous viscous representation is used in place of the inhomogeneous viscoplastic law of Eq. (3), using a matching technique [2, 3]
- (2) Our expressions relating generalized stresses (bending moments and in-plane forces) to corresponding strain rates are derived from a sandwich-plate model, with constants chosen to give correct uniform plate values for pure bending or extension.
- (3) Strain hardening is not considered explicitly in the constitutive equations.



- (4) Strain rate history effects are ignored, the constitutive law of viscous or viscoplastic behavior being derived from tests at constant strain rate.
- (5) Elastic deformations are neglected.
- (6) The pressure pulse on the specimen plate is idealized as impulsive, having zero duration.
- (7) In the extended mode technique the mode form solution appropriate to a deflection field is found, and the acceleration is found from this field; successive mode fields are continuous only at one point of the structure.
- (8) The numerical determination of mode shapes and accelerations (a non-linear eigenproblem) required iterations in our calculations, either in a direct iterative scheme or as part of a finite element method. Convergence was rapid at small deflections but was slow or non-existent at sufficiently large deflections. Although the computations up to the maximum deflections in the tests are believed very close to convergence, there remains some question as to accuracy. The finite element form of course involves additional uncertainties.

These errors are discussed in more detail elsewhere [4]. Arguments can be given that, apart from the "intrinsic" error and the last one in the above list, they are all positive, i.e., lead to an over-estimate of final midpoint deflection. Thus, test deflections falling below the estimate curves, as in Figs. 10, 11, 13-15 are in accord with these expectations.

To conclude, the test results indicate that (a) the further idealizations and approximations used in the mode technique are predominately conservative, as expected; (b) these outweigh the "intrinsic" error discussed above; and (c) pull-in at the clamped edge, occurring mainly at the large loads used with

$a/R = 1$ , and causing the actual deflections for these tests to exceed those expected for the fully fixed edge condition, offset the conservative idealizations and approximations of the mode technique in these tests. The present results do show the conservative nature of the deflection estimates by the mode technique when the prescribed constraint conditions are reasonably well satisfied, i.e., the tests for  $a/R < 1$ . The close agreement between the experiments and the predictions for  $a/R = 1$  seems a consequence of edge slippage effects counterbalancing the conservative features of the mode approximation method. A possibly better experimental arrangement to obtain fuller constraint against in-plane edge displacements might be to machine the plate specimen and clamping from a single block of metal.

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Table 2

## STEEL PLATES

$$\frac{a}{R_0} = \frac{1}{3}$$

(Buffer pad approx. 0.1 g.)

Test No.	Explosive		Plate Thickness H in.	Impulse I lb-sec	Final Mid-Point Deflection $w_o^f$ in.	Peak Time	Intercept Time	$\frac{f}{w_o} \frac{H}{H}$
	Thick-ness in.	Weight grams				$t_f'$ $\mu$ s	$t_f''$ $\mu$ s	
42	0.031	0.45	0.0760	0.191	0.100	Not Measured		1.32
45	0.058		0.0761	0.363	0.213	Not Measured		2.80
48	0.079	0.975	0.0760	0.443	0.264	105	75	3.47
49	0.031	0.45	0.0761	0.197	0.100	100	70	1.31
50	0.062	0.9	0.0762	0.379	0.223	105	70	2.93
51	0.079	0.975	0.0761	0.425	0.269	100	70	3.53
52	0.030	0.45	0.0761	0.186	0.081	100	70	1.06
53	0.062	0.9	0.0761	0.369	0.222	100		2.92
54	0.079	0.975	0.0761	0.435	0.264	105	70	3.47

## STEEL PLATES

$$\frac{a}{R_0} = \frac{1}{2}$$

(Buffer pad approx. 0.2 g.)

47	0.08	2.1	0.0761	0.884	0.448	-	-	5.89
62	0.079	2.0	0.0760	0.814	0.429	100	-	5.64
63	0.062	1.7	0.0761	0.706	0.364	-	-	4.78
64	0.062	1.7	0.0761	0.728	0.381	105	-	5.00
65	0.031	1.0	0.0760	0.371	0.183	100	-	2.41
66	0.031	0.95	0.0761	0.386	0.196	110	75	2.58
67	0.079	1.95	0.0760	0.828	0.425	105	-	5.59
68	0.079	1.95	0.0761	0.828	0.418	100	-	5.49
69	0.032	0.95	0.0761	0.395	0.201	-	-	2.64
70	0.079	2.0	0.0761	0.901	0.450	-	-	5.91
73	0.081	2.05	0.0761	0.853	0.451	-	-	5.92
44	0.058	1.6	0.0759	0.696	0.371	-	-	4.89



# STEEL PLATES

$$\frac{a}{R_o} = 1$$

(Buffer pad approx. 0.7 g.)

Test No.	Explosive		Plate Thickness H in.	Impulse I lb-sec	Final Mid-Point De-flection $w_o^f$ in.	Peak Time	Intercept Time	$\frac{f}{w_o} \frac{H}{H}$
	Thickness in.	Weight grams				$t_f' \mu s$	$t_f'' \mu s$	
46	0.031	4.0	0.0761	1.608	0.484	-	-	6.36
55	0.017	1.9	0.0761	0.707	0.197	120	-	2.59
56	0.025	3.1	0.0760	1.227	0.389	100	-	5.12
57	0.026	3.2	0.0761	1.257	0.384	105	80	5.05
58	0.017	1.9	0.0760	0.205	0.034	-	-	0.45
59	0.017	1.95	0.0760	0.714	0.208	115	90	2.74
60	0.021	2.5	0.0760	1.031	0.314	100	80	4.13
61	0.033	3.8	0.0761	1.577	0.486	100	-	6.38

# TITANIUM PLATES

$$\frac{a}{R_o} = \frac{1}{3}$$

(Buffer pad approx. 0.1 g.)

Test No.	Explosive		Plate Thickness H in.	Impulse I lb-sec	Final Mid-Point Deflection $w_o^f$ in.	Peak time $t_f'$ $\mu$ s	Intercept time $t_f''$ $\mu$ s	$\frac{w_o^f}{H}$
	Thickness in.	Weight grams						
84	0.078	0.9	0.0930	0.378	0.214	-	-	2.30
85	0.078	0.9	0.0930	0.402	0.241	-	-	2.59
86	0.079	0.975	0.0931	0.383	0.243	-	-	2.61
87	0.079	0.95	0.0938	0.368	0.221	-	-	2.36
88	0.079	0.95	0.0930	0.408	0.237	65	-	2.55
90	0.062	0.75	0.0923	0.311	0.191	-	-	2.07
91	0.061	0.75	0.0922	0.330	0.184	-	-	2.00
97	0.078	0.95	0.0928	0.398	0.260	68	-	2.80
98	0.060	0.8	0.0929	0.314	0.198	72	-	2.13
99	0.031	0.45	0.0930	0.192	0.110	70	-	1.18
100	0.108	1.4	0.0923	0.507	0.338	-	-	3.66
101	0.108	1.4	0.0896	0.472	0.350	-	-	3.90

$$\frac{a}{R_o} = \frac{1}{2}$$

(Buffer pad approx. 0.2 g.)

71	0.079	1.95	0.0933	0.900	0.436	65	-	4.67
72	0.078	2.0	0.0928	0.890	0.441	68	45	4.75
74	0.062	1.7	0.0927	0.705	0.362	72	50	3.91
75	0.062	1.7	0.0929	0.720	0.366	72	50	3.94
76	0.032	1.0	0.0930	0.384	0.182	65	50	1.96
77	0.030	1.0	0.0922	0.395	0.190	68	50	2.06

## TITANIUM PLATES

$$\frac{a}{R_o} = 1$$

(Buffer pad approx. 0.7 g.)

Test No.	Explosive		Thickness H in.	Impulse I lb-sec	Final Mid-Point De-flection $\frac{f}{w_o}$	Peak Time	Intercept Time	$\frac{f}{H}$
	Thickness in.	Weight grams				$t'_f$ $\mu$ s	$t''_f$ $\mu$ s	
78	0.031	3.2	0.0930	1.193	0.331	70	50	3.56
79	0.031	3.8	0.0924	1.601	0.446	-	-	4.83
80	0.025	2.8	0.0925	1.264	0.363	-	-	3.92
81	0.025	2.8	0.0921	1.232	0.351	-	-	3.81
82	0.017	2.1	0.0929	0.804	0.227	72	-	2.44
83	0.017	2.1	0.0929	0.334	0.083	-	-	0.89
89	0.031	3.8	0.0930	1.493	0.438	-	-	4.71
92	0.030	3.5	0.0928	1.347	0.403	66	48	4.34
93	0.031	3.6	0.0930	1.438	0.427	66	48	4.59
94	0.024	2.7	0.0931	1.069	0.315	68	-	3.38
95	0.017	1.95	0.0921	0.816	0.235	72	-	2.55
96	0.033	3.8	0.0918	1.600	0.499	64	45	5.43

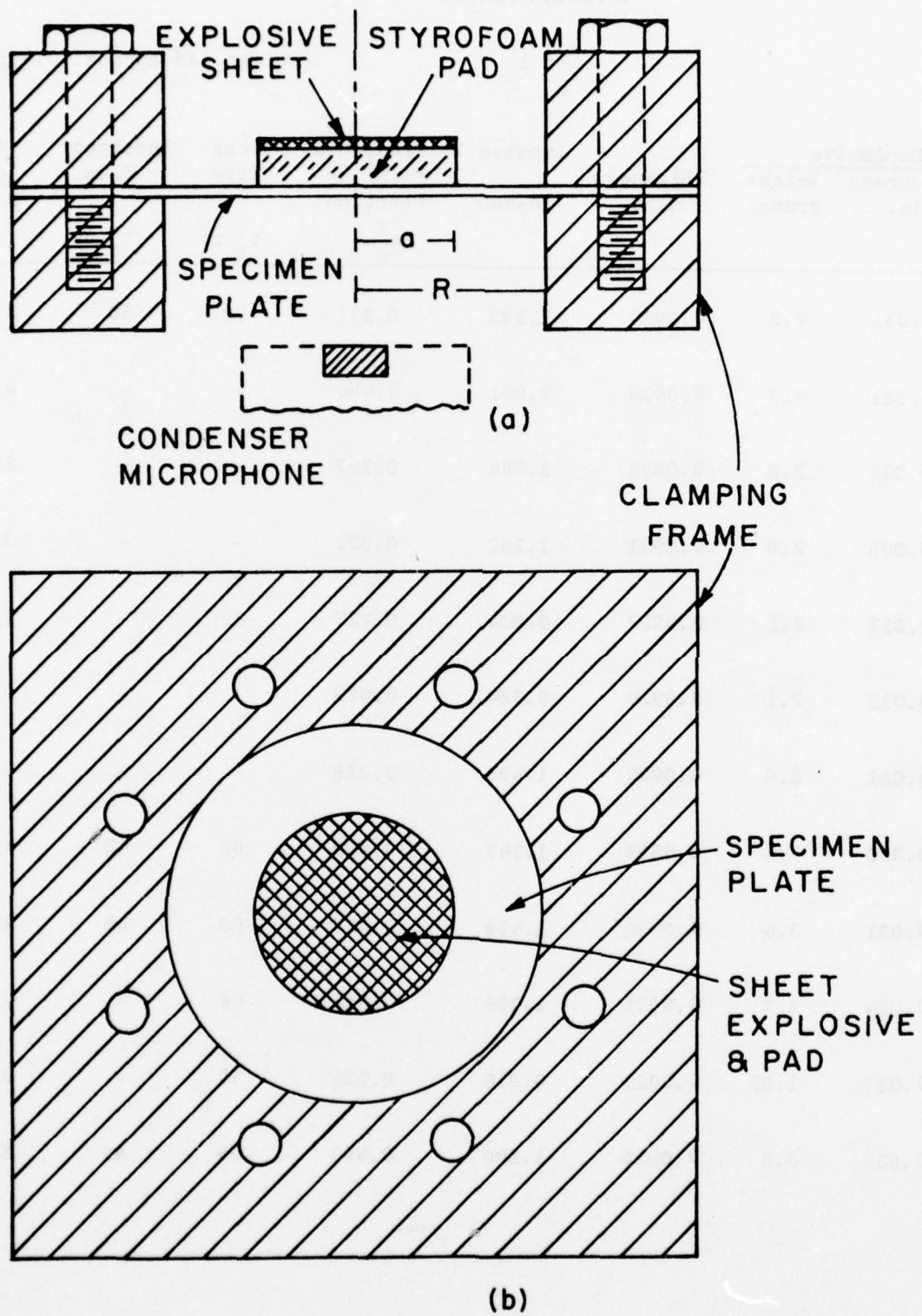


FIG. 1



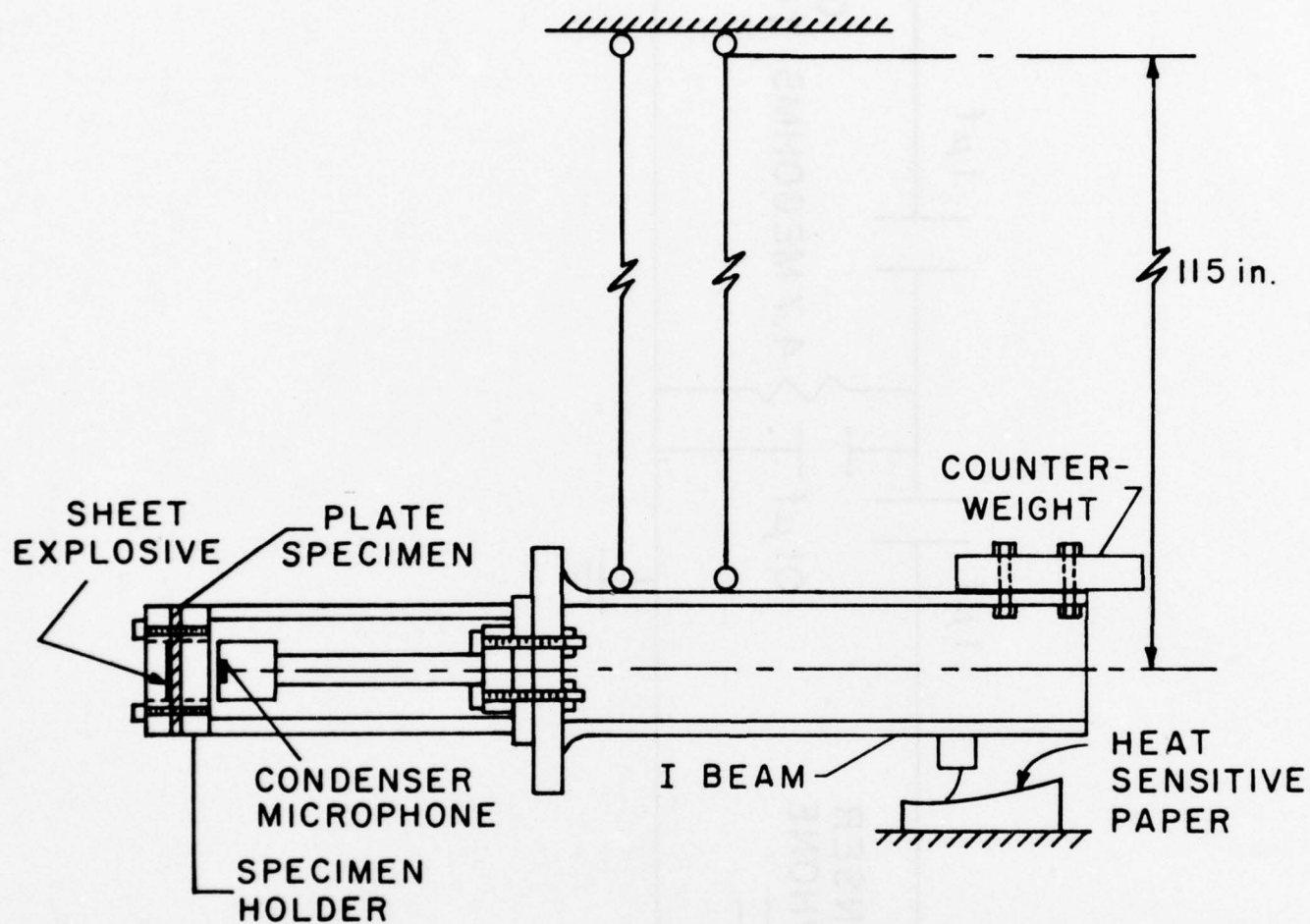


FIG. 2

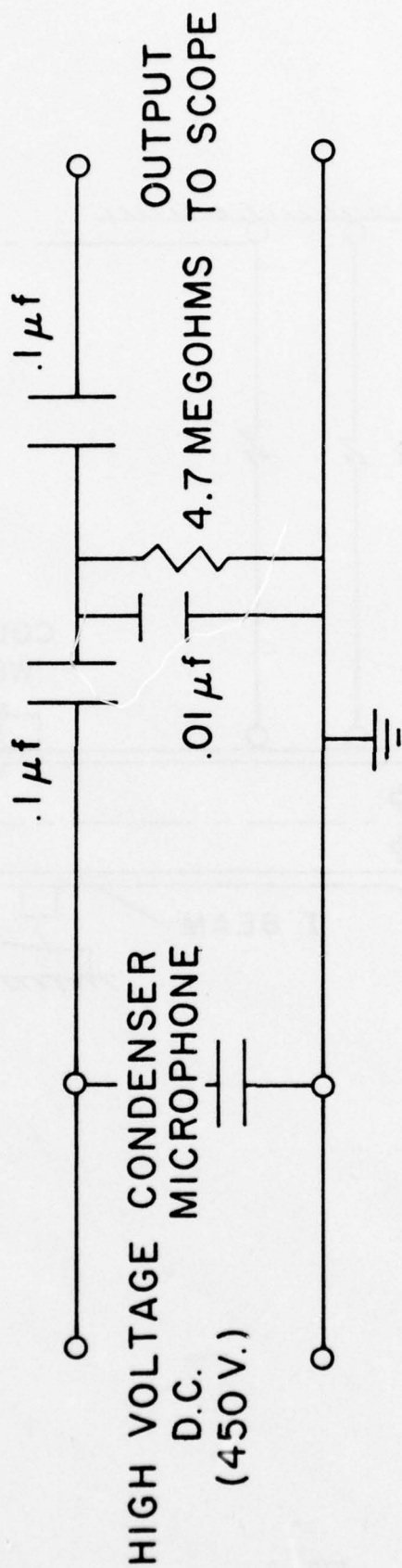
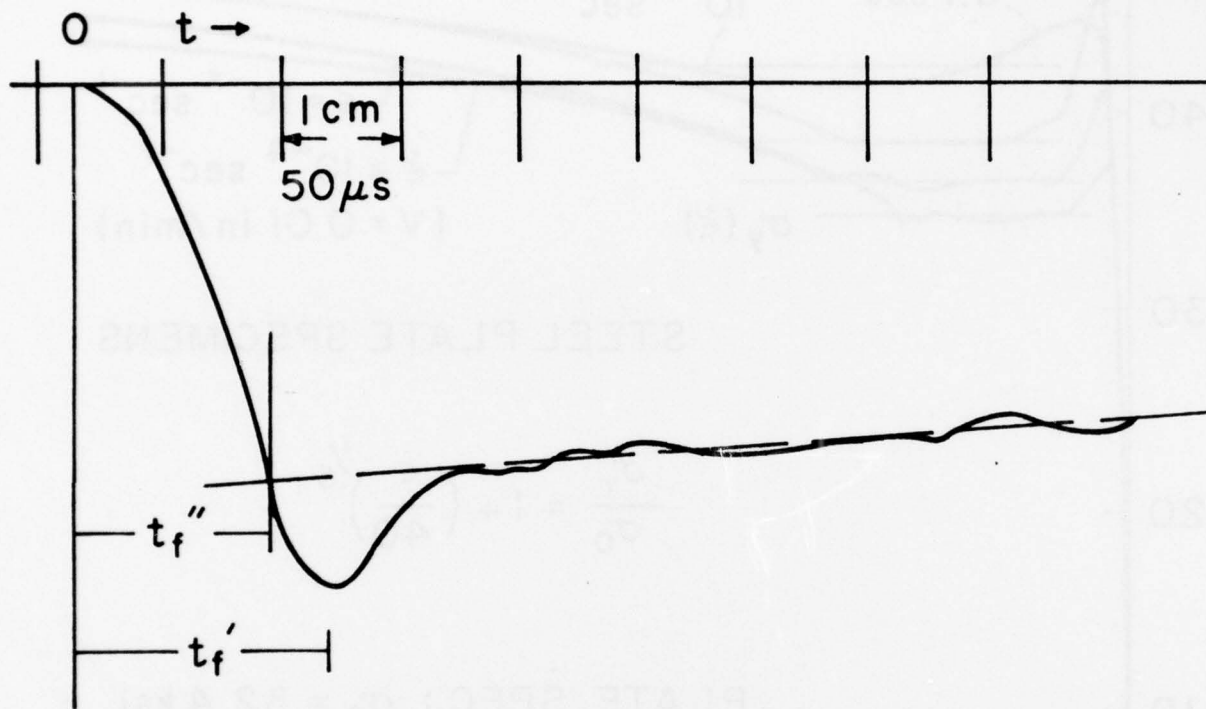


FIG. 3



TEST 57 CONDENSER MICROPHONE  
STEEL PLATE  $\frac{a}{R} = 1$ ,  $w_f = 0.384 \text{ in.}$

$$t_f'' \approx 2.1 \text{ cm} \times 50 = 110 \mu\text{sec}$$

$$t_f' \approx 1.6 \text{ cm} \times 50 = 80 \mu\text{sec}$$

FIG. 4

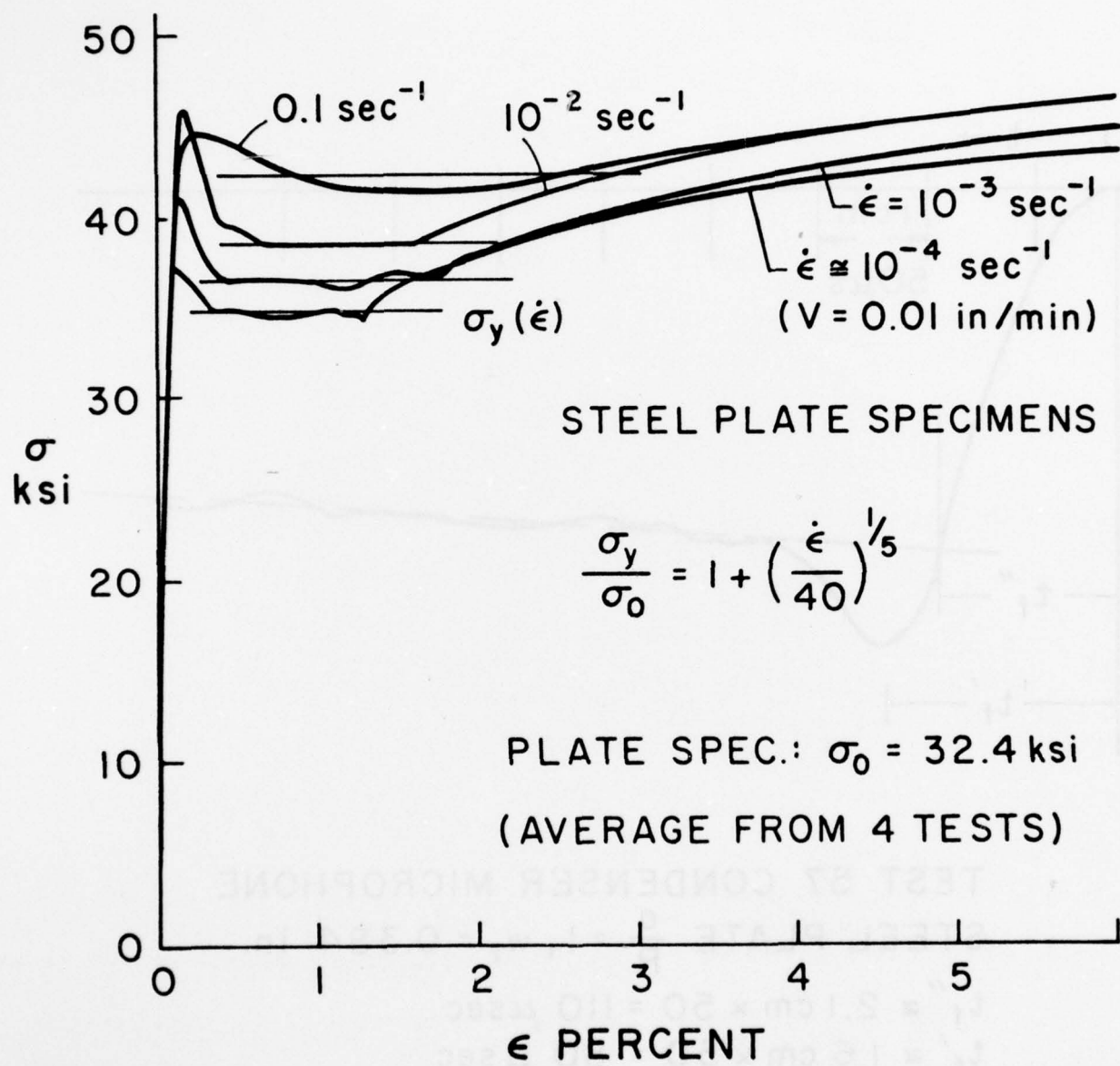


FIG. 5



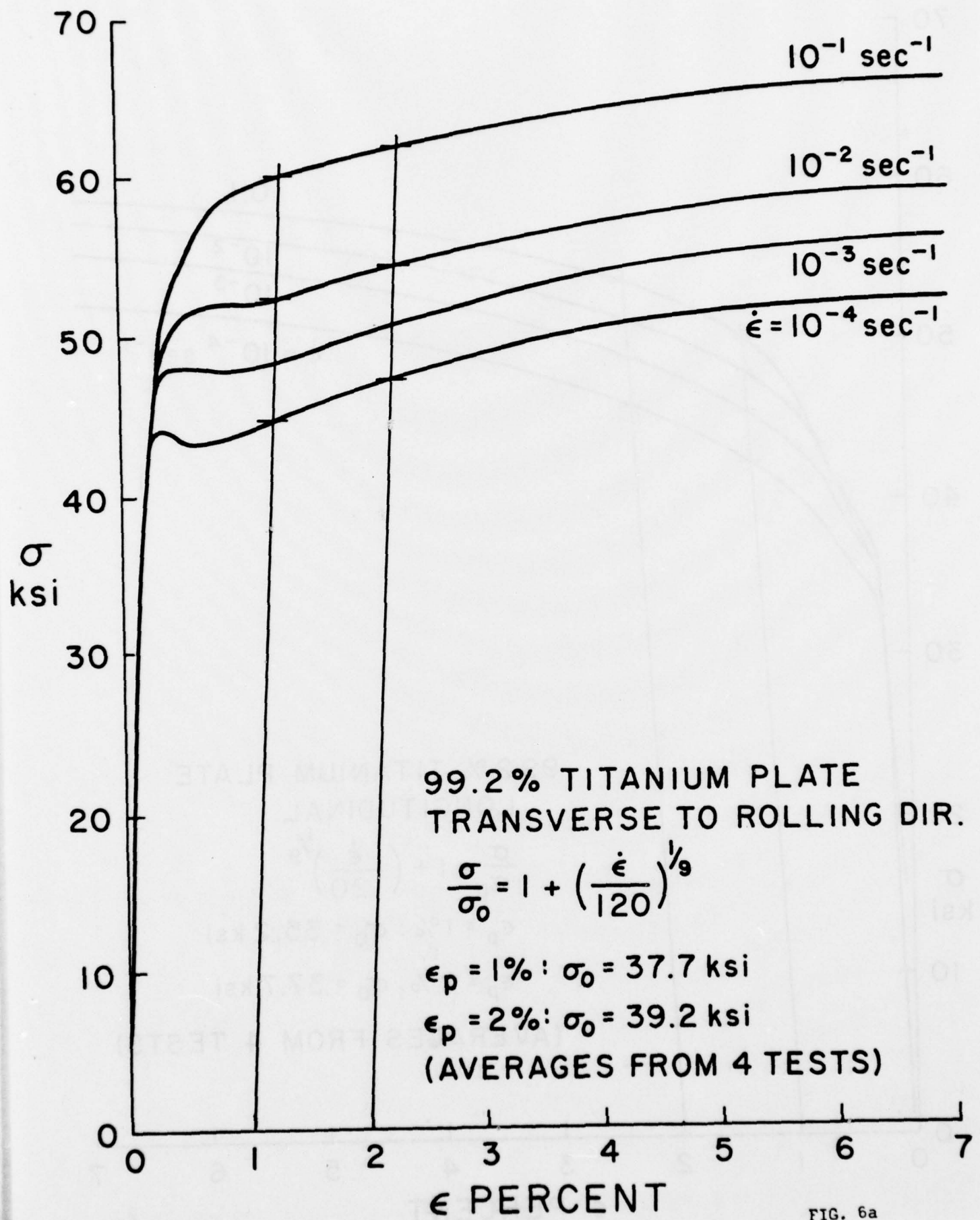


FIG. 6a

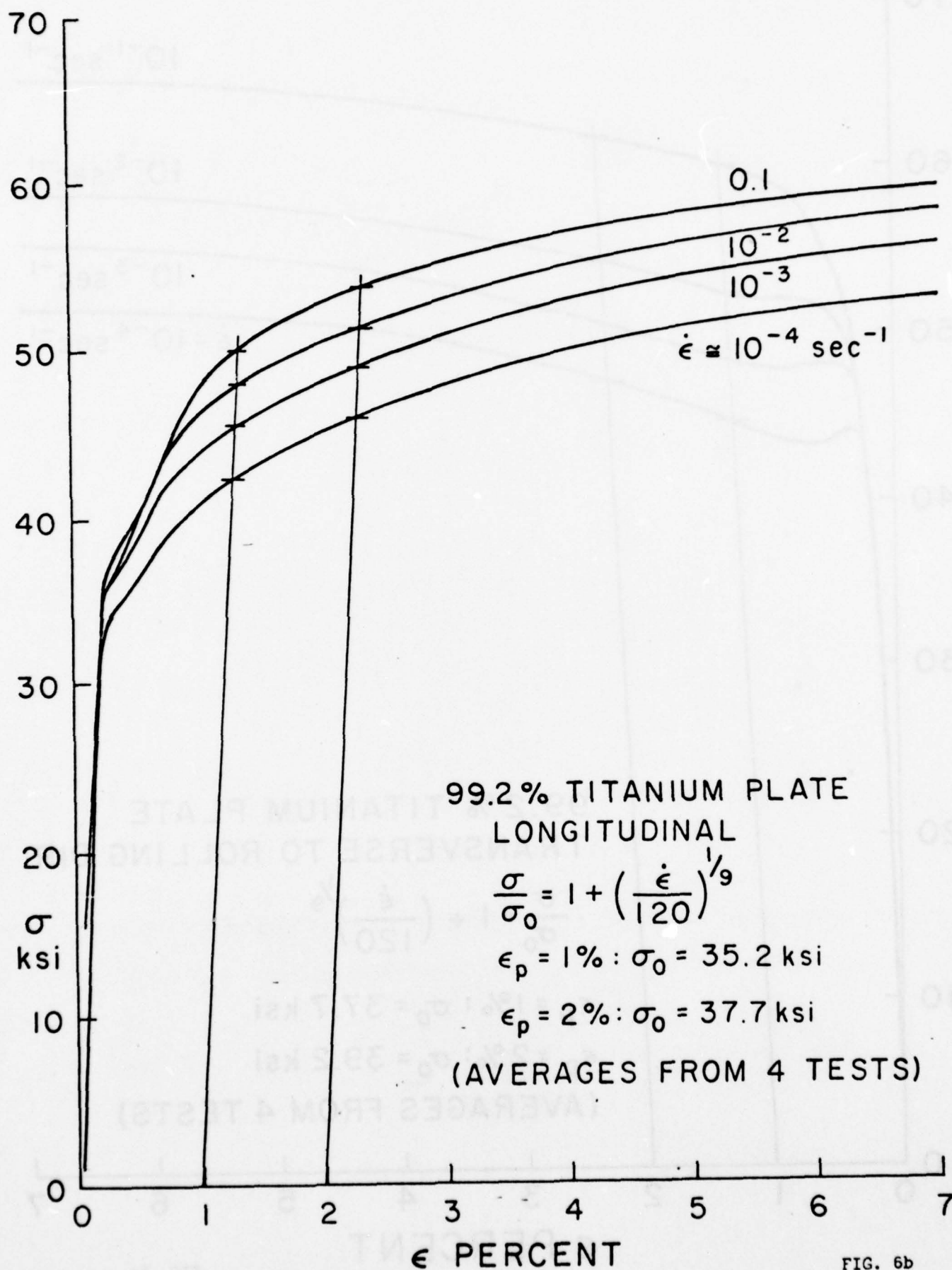


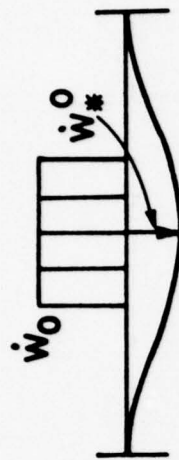
FIG. 6b

# TEST 51 STEEL PLATE

$$\frac{a}{R} = \frac{1}{3} \quad I = 0.425 \text{ lb-sec}$$

$$W_0 = 0.269 \text{ in.}$$

$$\text{MODE APPROX. } W_*^f = 0.332 \text{ in., } \frac{a}{R} = \frac{1}{3}$$



$$\dot{w}_*^0 = 0.578 \dot{w}_0$$

$$\frac{a}{R} = \frac{1}{3}$$

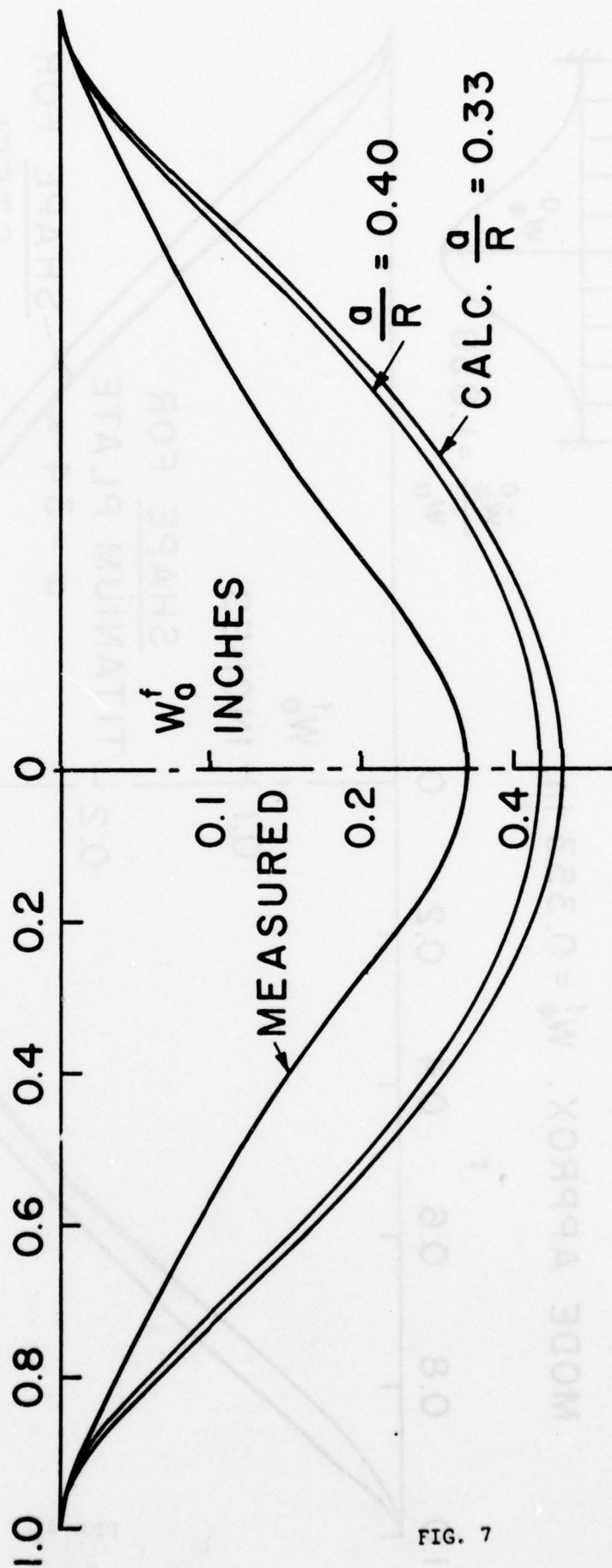


FIG. 7

# TEST 57: STEEL PLATE $\frac{a}{R} = 1$

$I = 1.26 \text{ lb-sec}$   $W_0^f = 0.384 \text{ in.}$

MODE APPROX.  $W_*^f = 0.353 \text{ in.}$

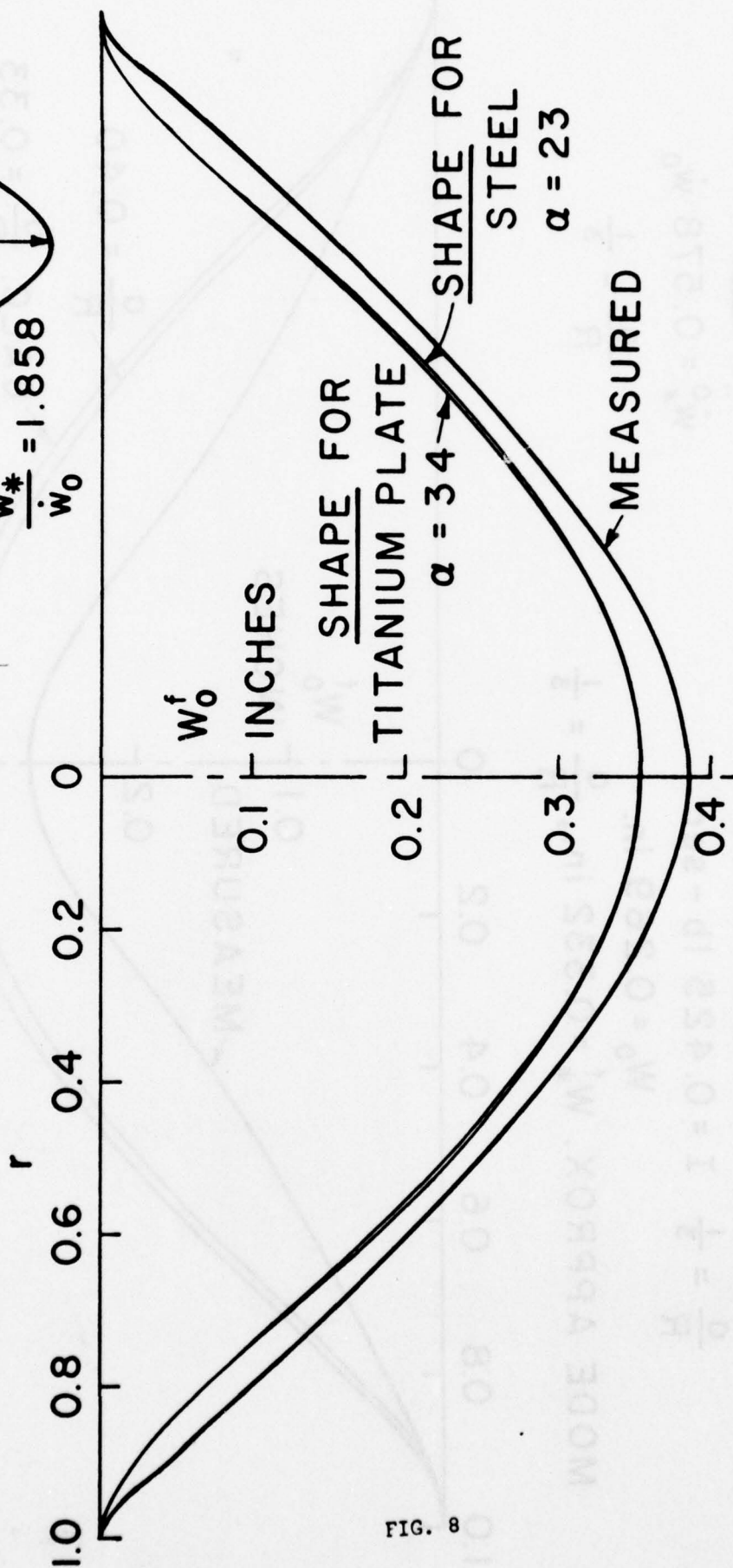
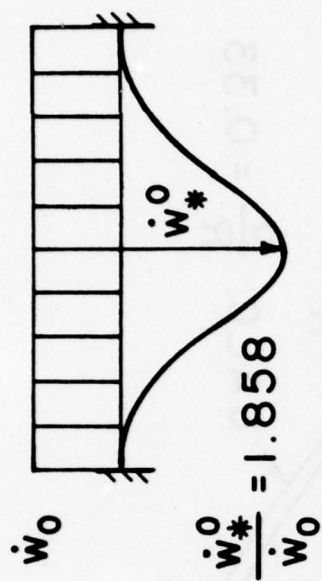
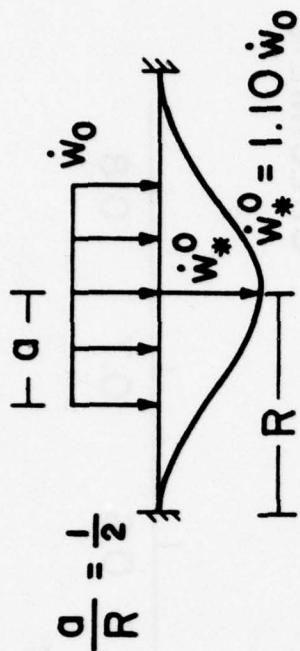
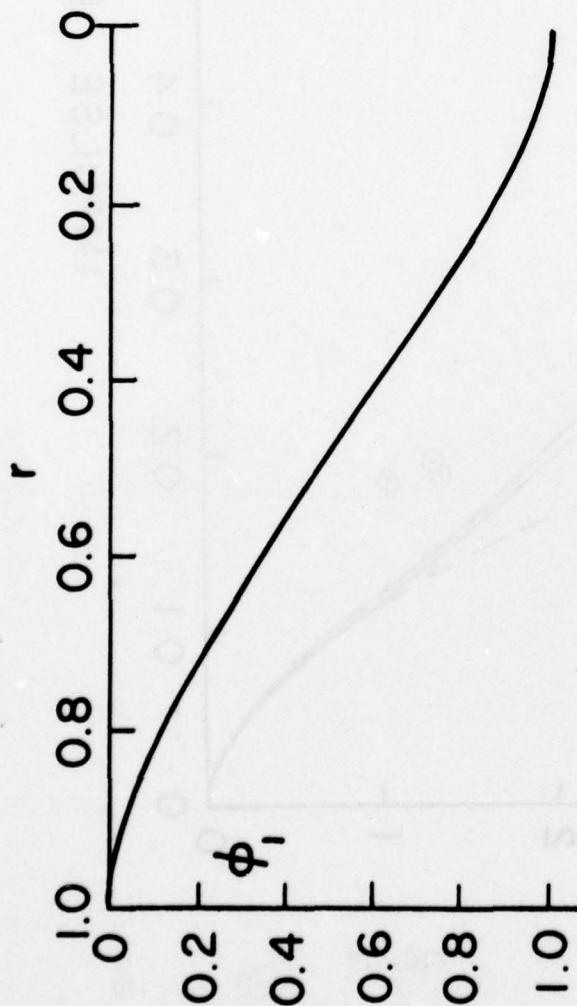


FIG. 8





TEST T74 TITANIUM PLATE  
 $I = 0.705 \text{ lb-sec } w_0^f = 0.36 \text{ in.}$

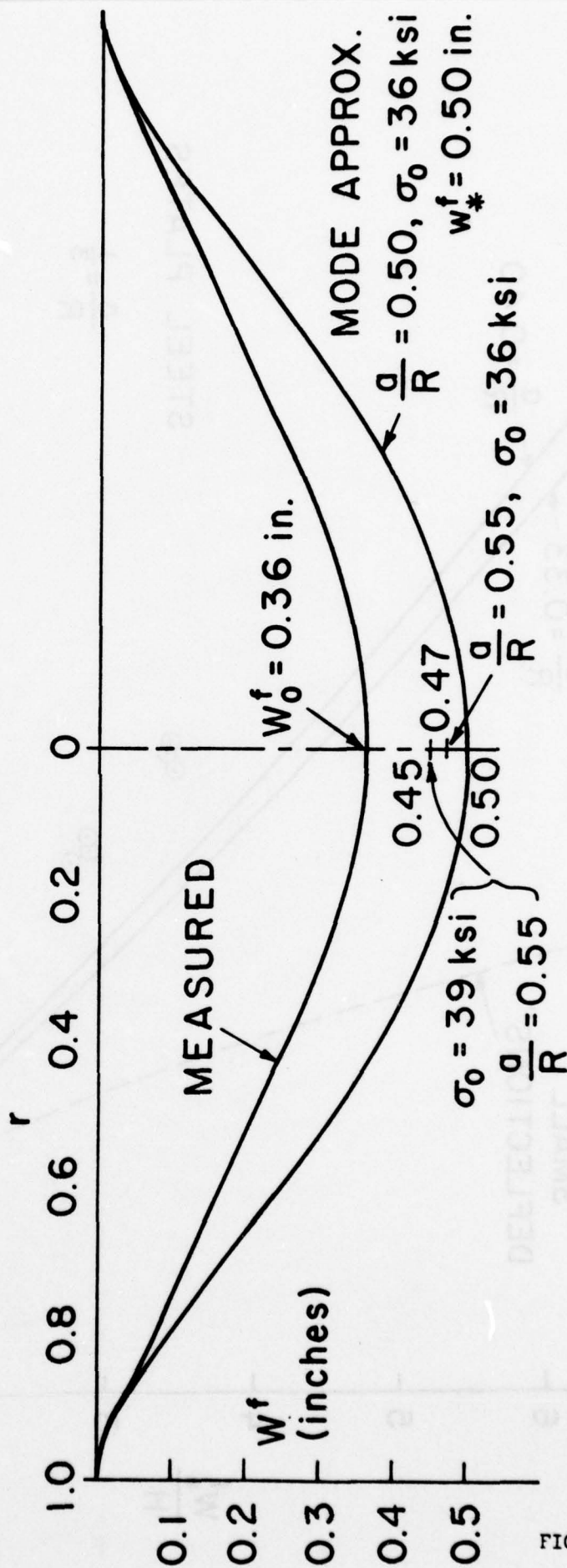


FIG. 9

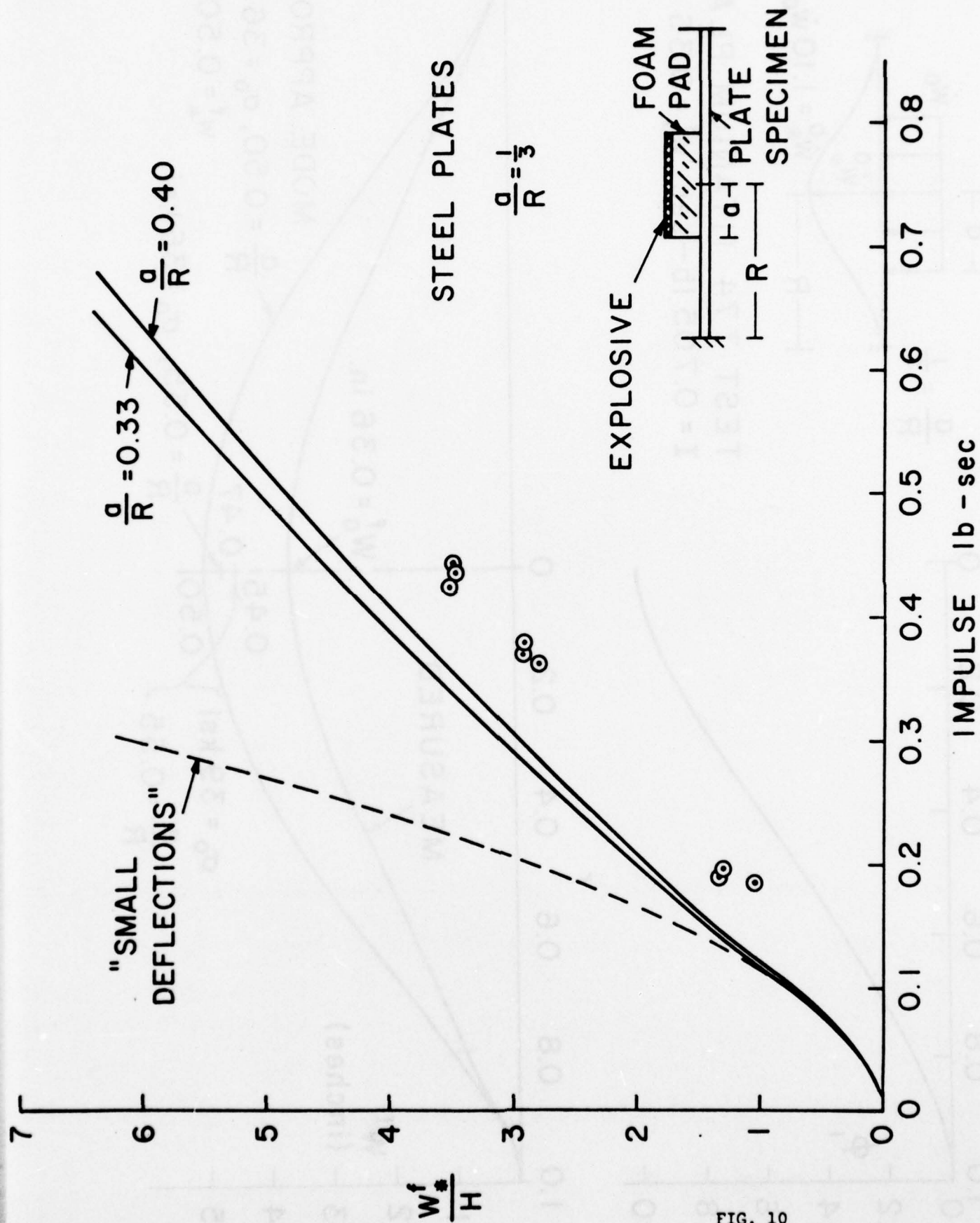


FIG. 10

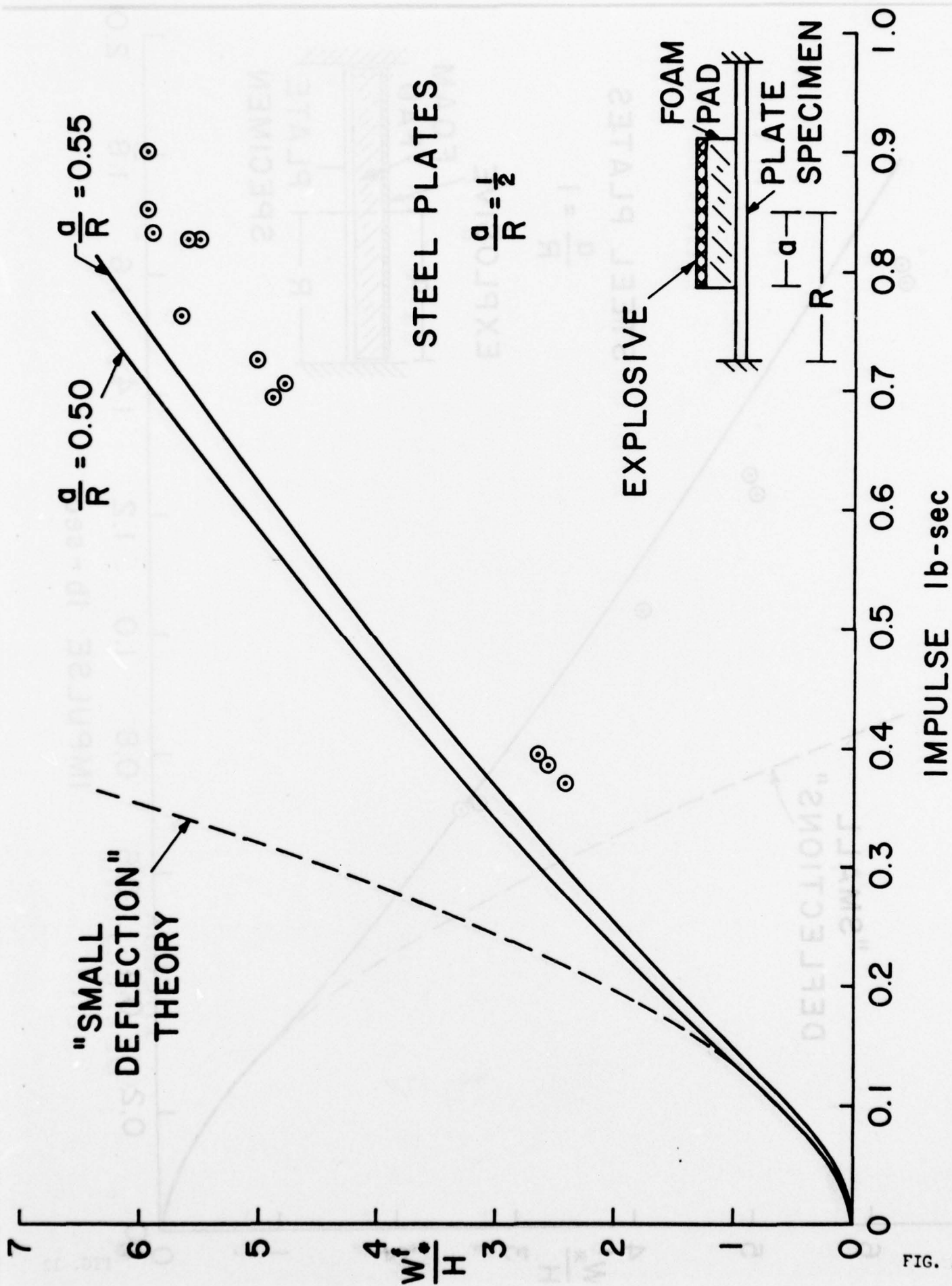


FIG. 11

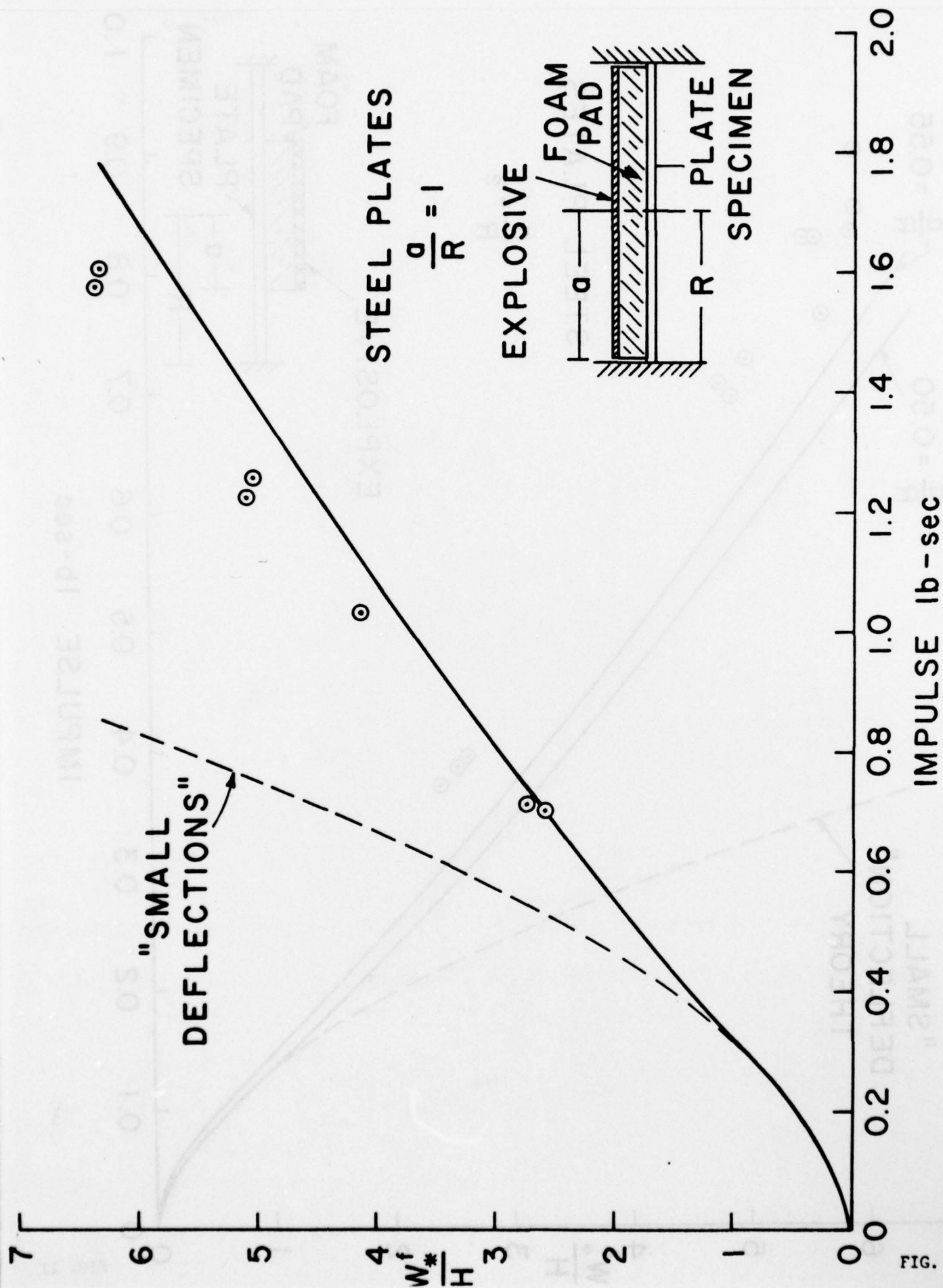


FIG. 12



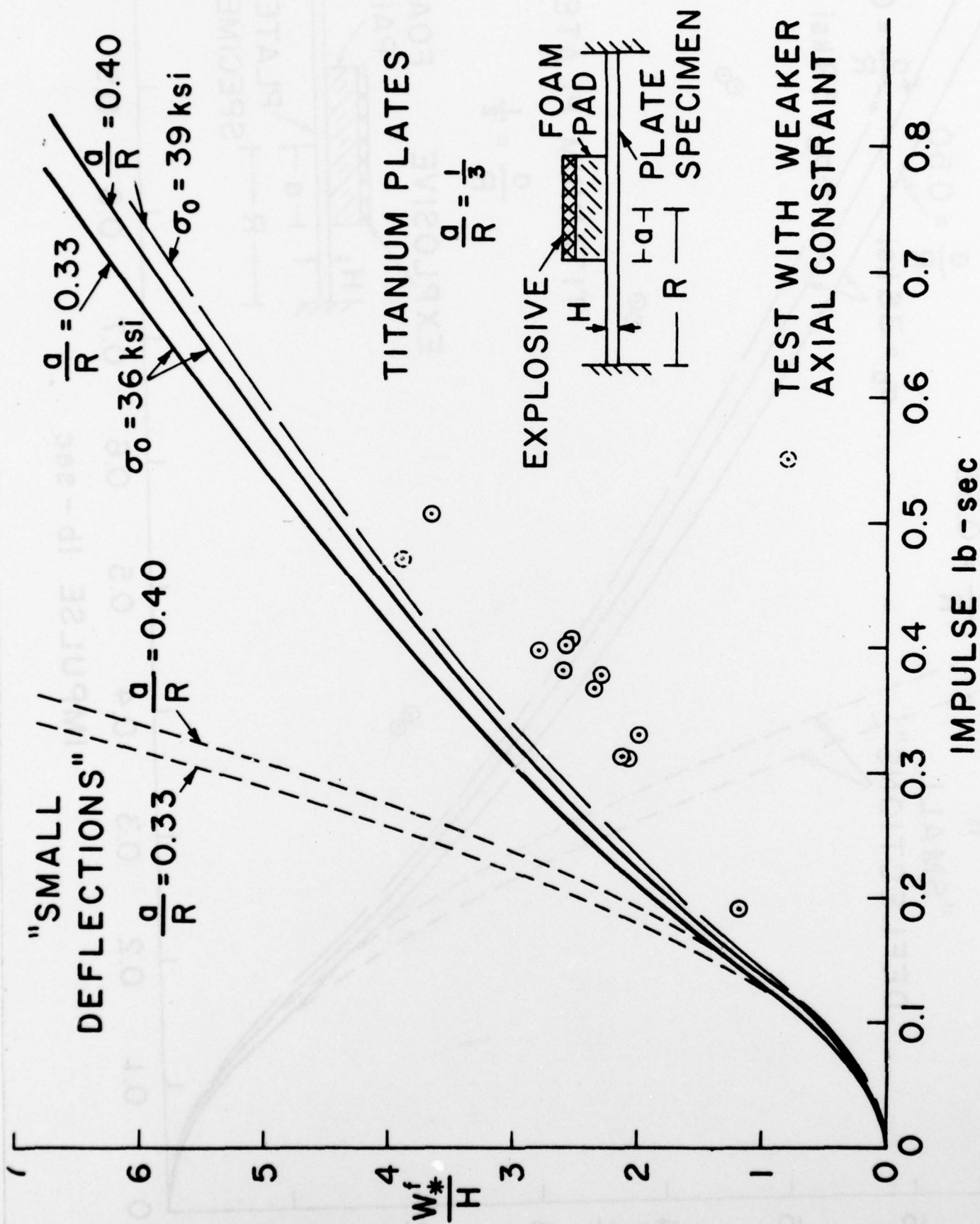


FIG. 13

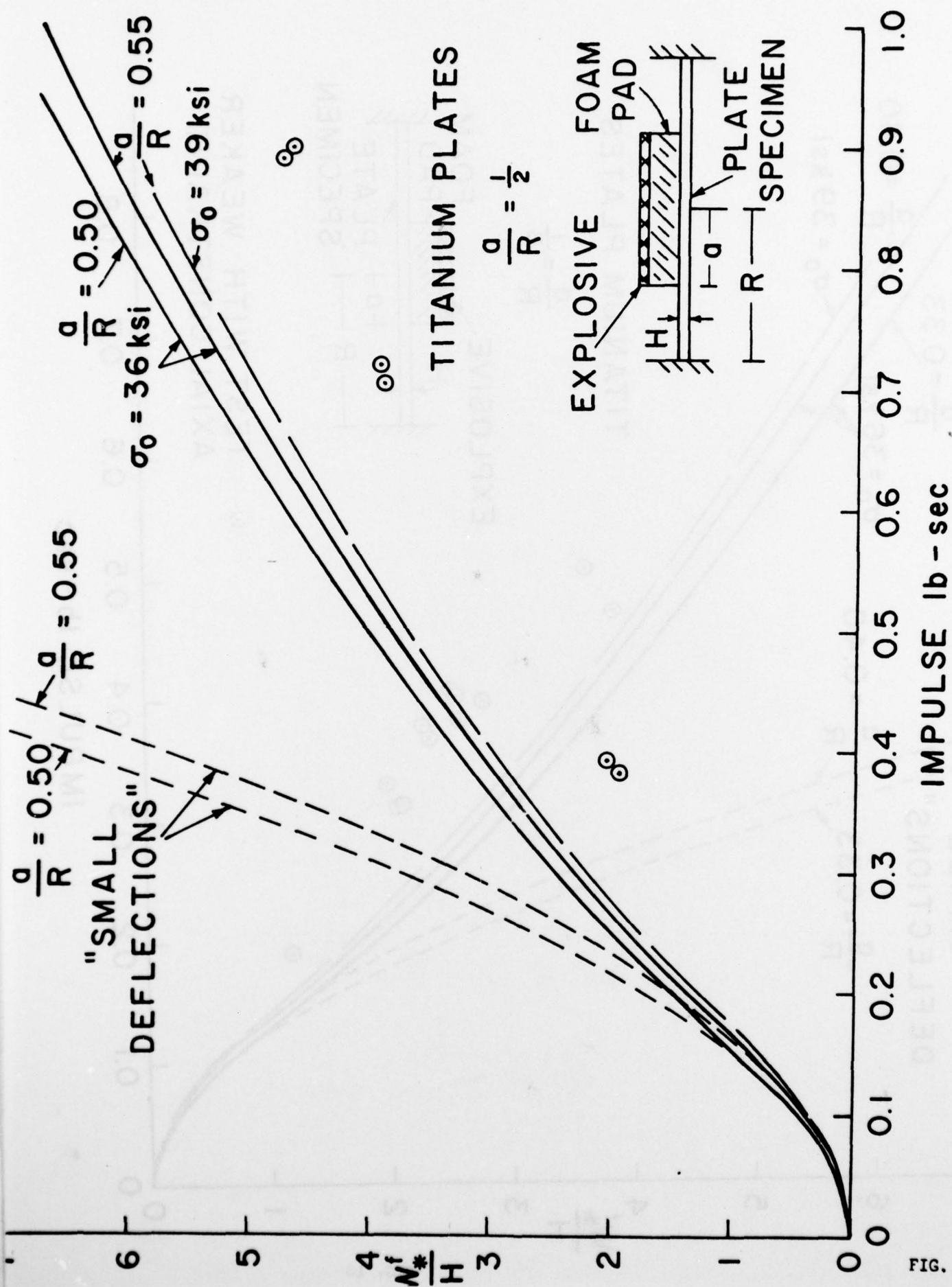


FIG. 14

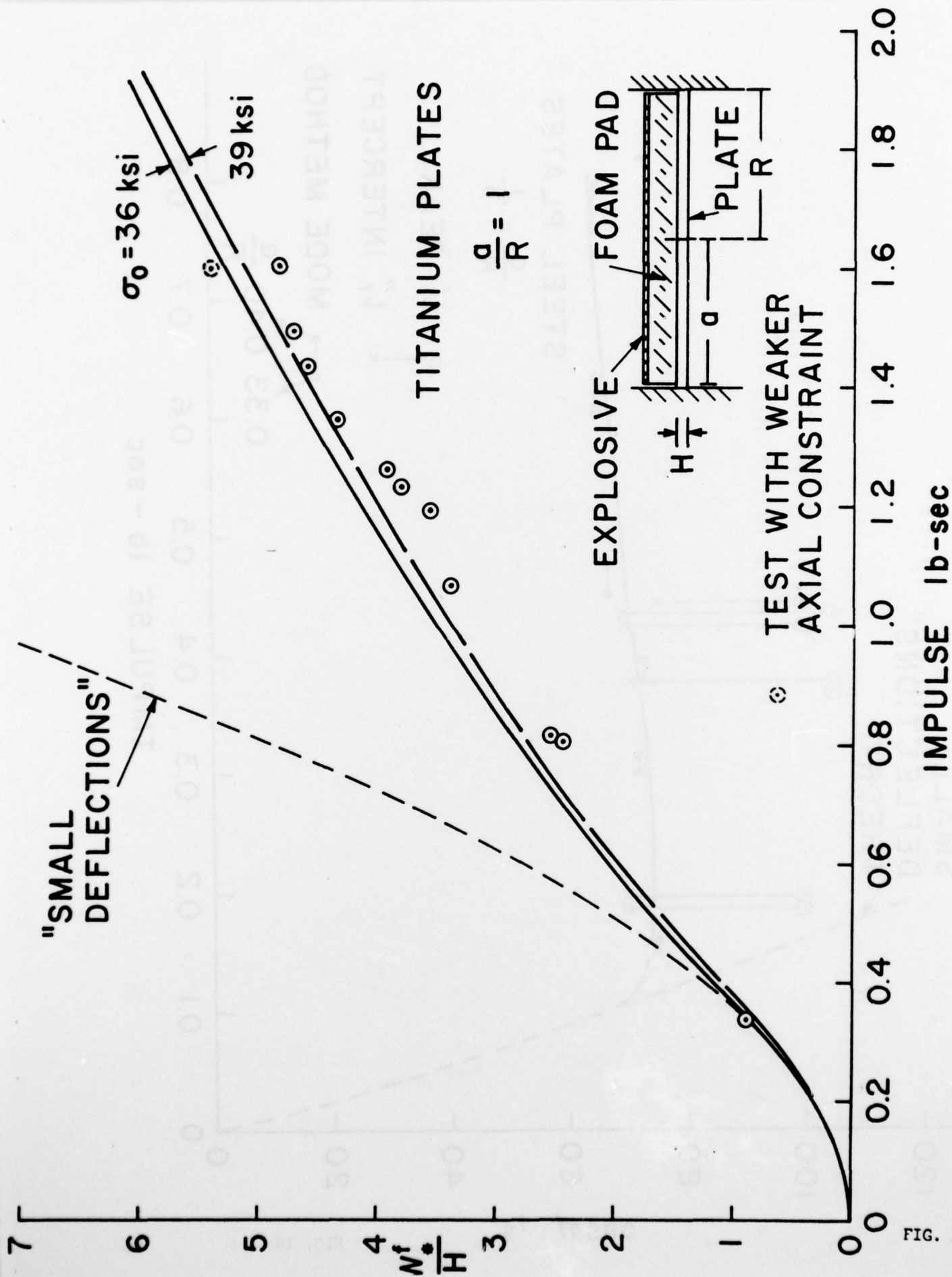


FIG. 15

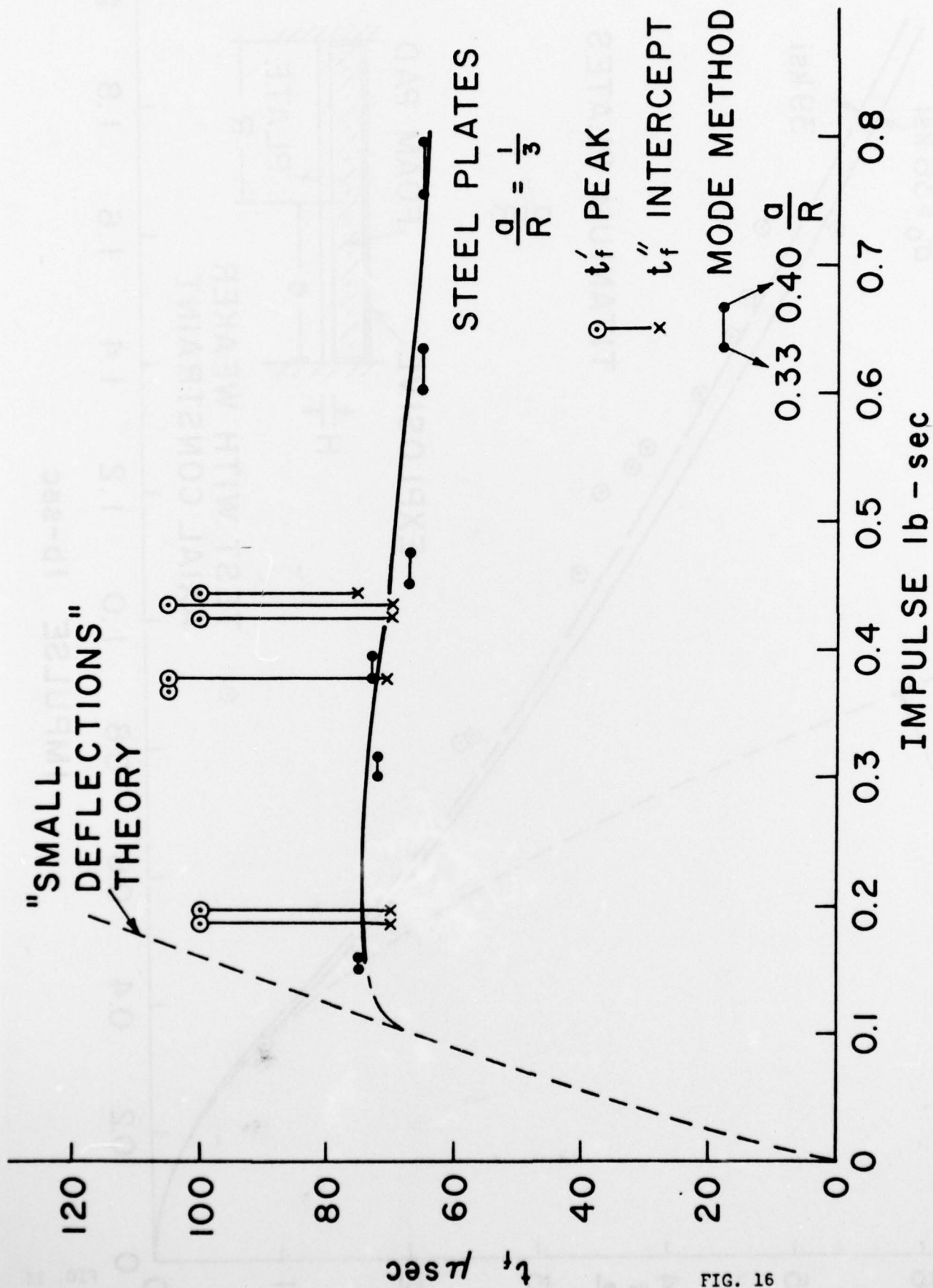


FIG. 16



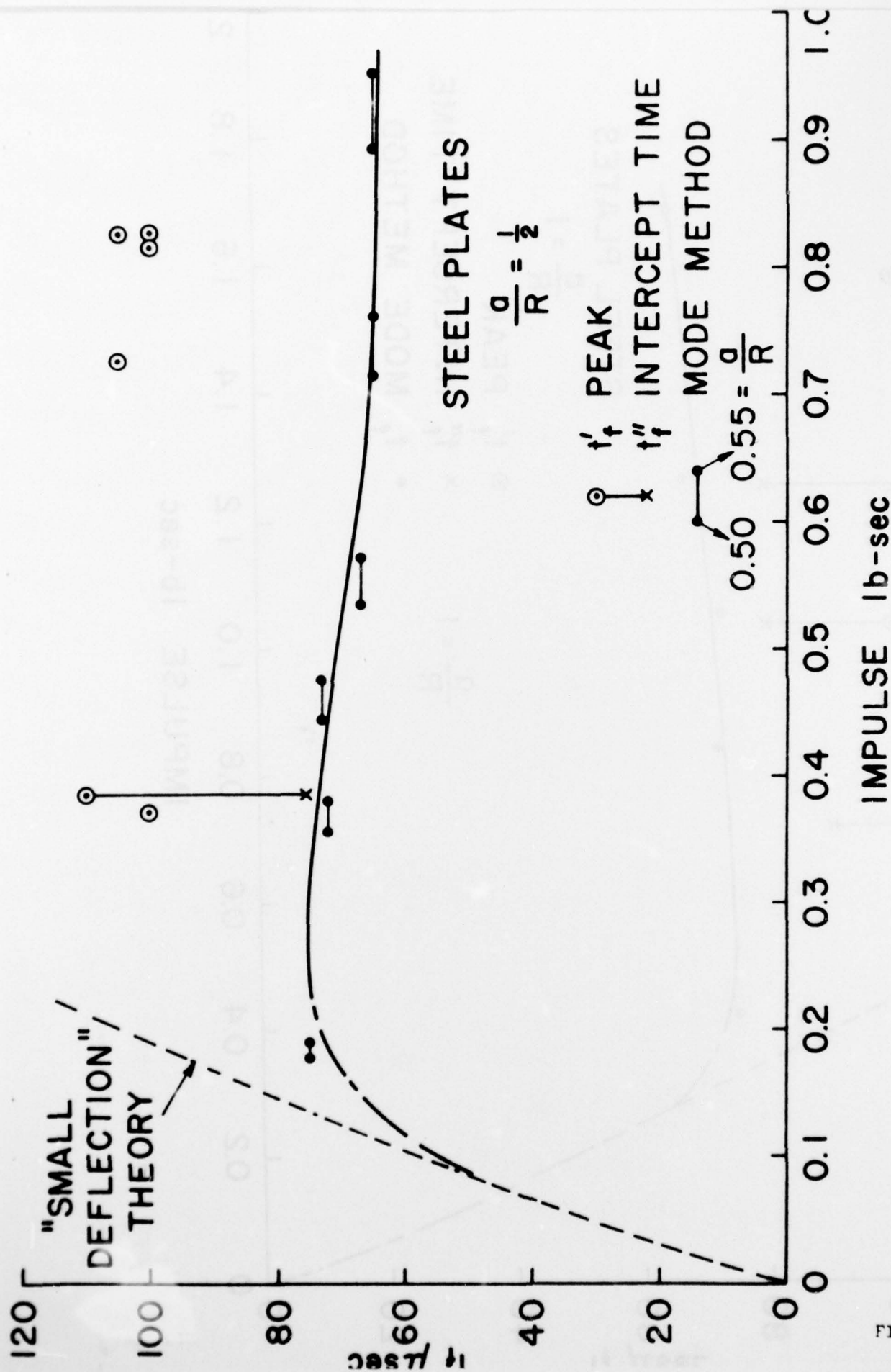


FIG. 17

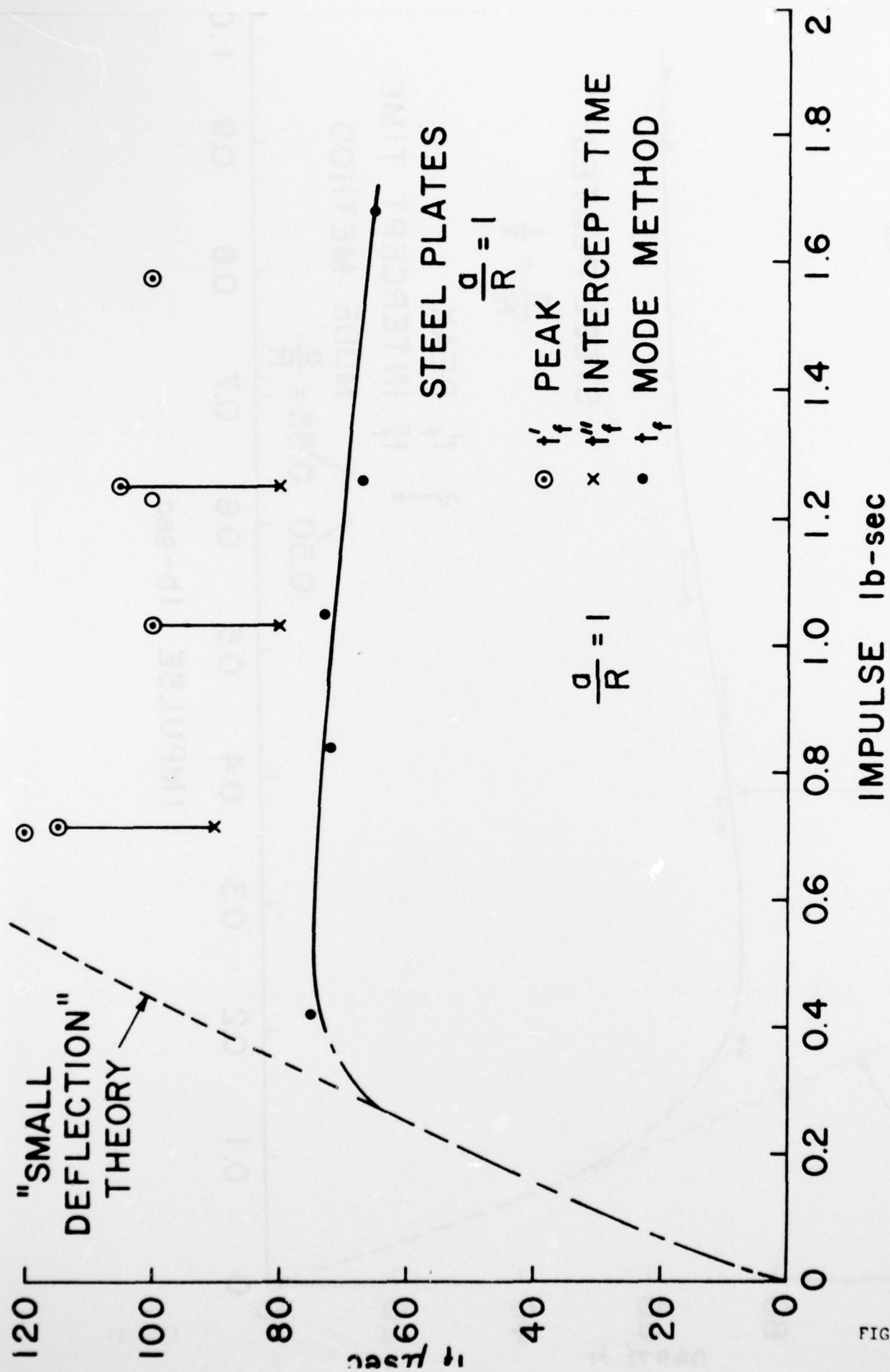


FIG. 18

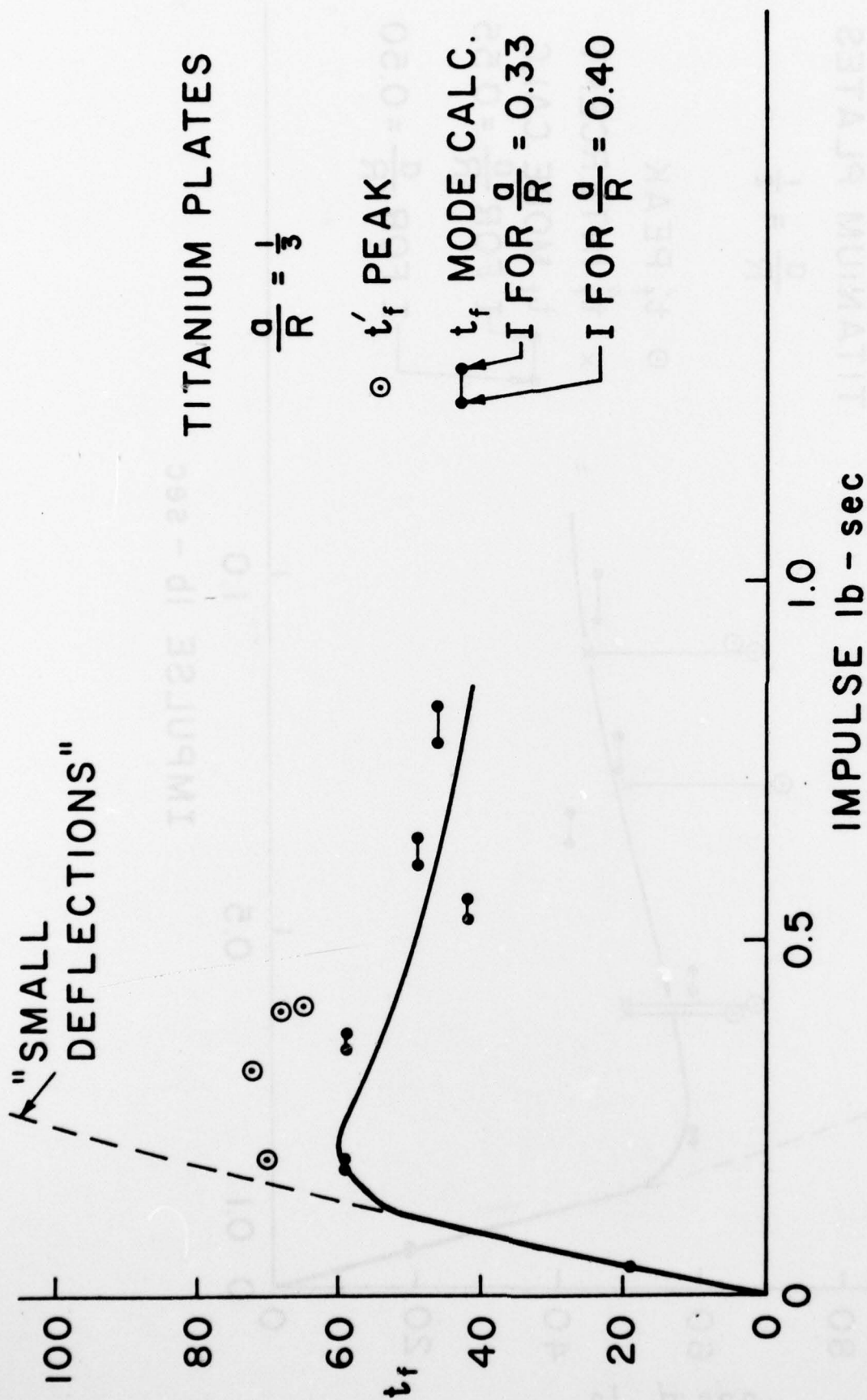


FIG. 19

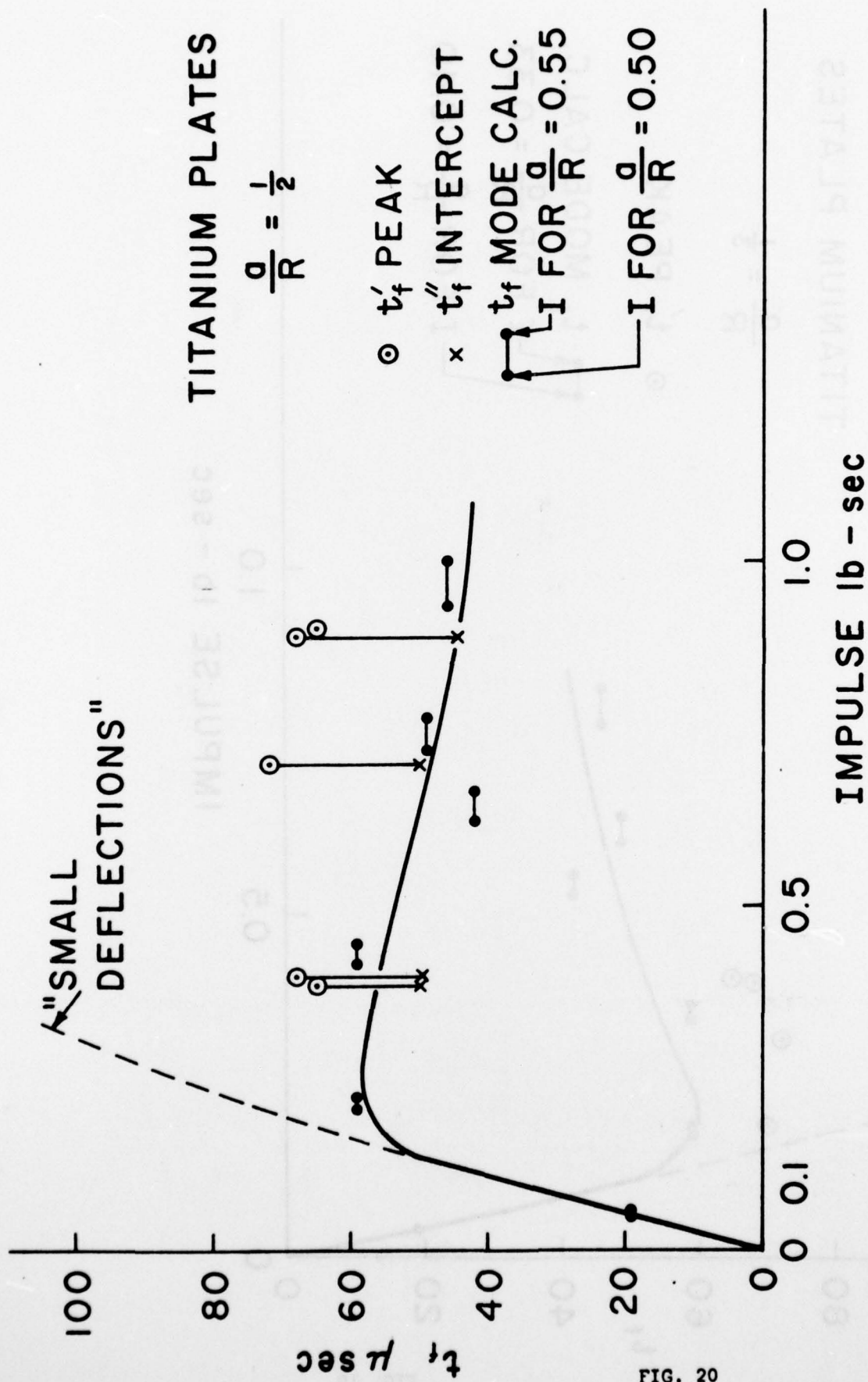


FIG. 20



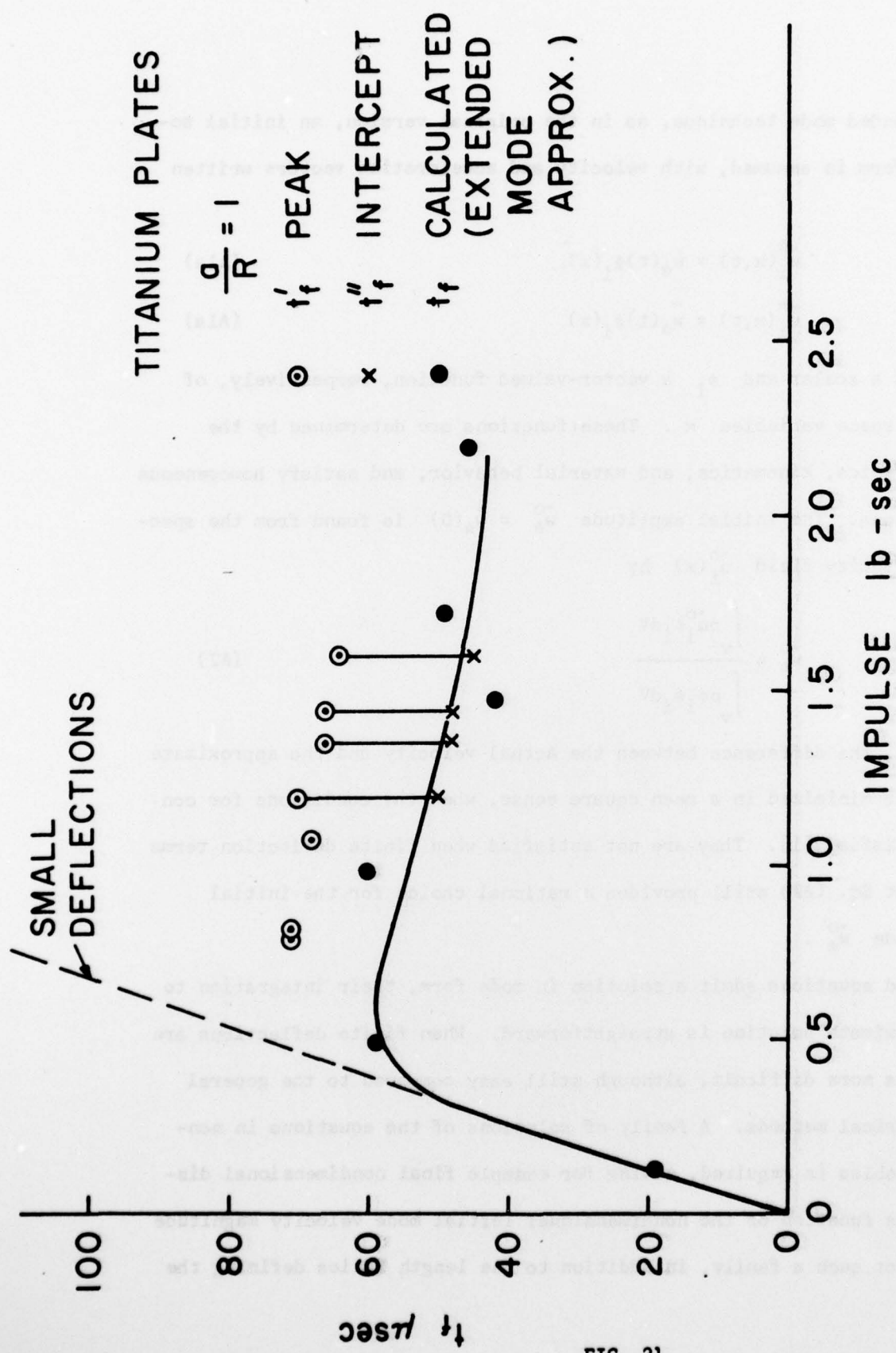


FIG. 21

## Appendix

In the extended mode technique, as in the original version, an initial solution in mode form is assumed, with velocity and acceleration vectors written as

$$\dot{u}_i^*(x, t) = \dot{w}_*(t) \phi_i(x) \quad (A1a)$$

$$\ddot{u}_i^*(x, t) = \ddot{w}_*(t) \phi_i(x) \quad (A1a)$$

where  $\dot{w}_*(t)$  is a scalar and  $\phi_i$  a vector-valued function, respectively, of time  $t$  and of space variables  $x$ . These functions are determined by the equations of dynamics, kinematics, and material behavior, and satisfy homogeneous boundary conditions. The initial amplitude  $\dot{w}_*^0 = \dot{w}_*(0)$  is found from the specified initial velocity field  $\dot{u}_i^0(x)$  by

$$\dot{w}_*^0 = \frac{\int_V \rho \dot{u}_i^0 \phi_i dV}{\int_V \rho \phi_i \phi_i dV} \quad (A2)$$

With this choice, the difference between the actual velocity and the approximate field Eq. (A1) is minimized in a mean square sense, when the conditions for convergence are satisfied [1]. They are not satisfied when finite deflection terms are included, but Eq. (A2) still provides a rational choice for the initial velocity amplitude  $\dot{w}_*^0$ .

If the field equations admit a solution in mode form, their integration to obtain the approximate solution is straightforward. When finite deflections are considered, it is more difficult, although still easy compared to the general solution by numerical methods. A family of solutions of the equations in non-dimensional variables is required, giving for example final nondimensional displacement  $w_*^f$  as function of the nondimensional initial mode velocity magnitude  $\dot{w}_*^0$ . To construct such a family, in addition to the length ratios defining the

geometry of the structure one must specify parameters of material behavior. For a plate of rate sensitive behavior, these can be taken as

$$n ; \quad \alpha = \frac{8R_o^3}{H^2} \dot{\epsilon}_o \sqrt{\frac{2\rho}{\sigma_o}} \quad (A3a,b)$$

where  $\sigma_o$ ,  $\dot{\epsilon}_o$ ,  $n$  are the material constants appearing in the stress-strain expression Eq. (3),  $\rho$  is the mass per unit volume,  $R_o$  and  $H$  are the radius and the thickness of the plate, respectively. Thus the solution of the mode problem furnishes the final deflection and response time, as

$$w_{\#}^f = F_1(\dot{w}_{\#}^o, \alpha, n, R_o/H) \quad (A4a)$$

$$t_f = F_2(\dot{w}_{\#}^o, \alpha, n, R_o/H) \quad (A4b)$$

where  $w_{\#}^f = w_{\#}(t_f)$  is final deflection,  $t_f$  is time at which the structure started in mode motion with velocity  $\dot{w}_{\#}^o$  comes to rest; all quantities are non-dimensional. To obtain the relation between measured impulse (in physical units) and final deflection or response time we need the relation between  $\dot{w}_{\#}^o$  and  $\dot{w}_o$ , the nondimensional initial velocity imparted by impulsive pressure, assumed constant over area of radius  $a$ . The general mode velocity equation (A2) gives, for our plate problem

$$\frac{\dot{w}_{\#}^o}{\dot{w}_o} = \frac{\int_0^a \phi_1^2 r dr}{\int_0^{R_o} \phi_1^2 r dr} = J\left(\frac{a}{R_o}\right) \quad (A5)$$

where  $r$  is the radial coordinate and  $\phi_1(r)$  is the initial shape function. The relation between  $\dot{w}_o$  and test impulse  $I$  is

$$\dot{w}_o = \frac{\tau}{H} V_o = \frac{\tau}{H} \frac{I}{\pi a^2 H \rho} \quad (A6)$$

where  $\tau$  is a constant with dimensions of time. Hence

$$I = \dot{w}_o^* \frac{\pi R_o^2 H^2 \rho a^2 / R_o^2}{\tau J(a/R_o)} = \dot{w}_o^* \left( \frac{\pi}{\sqrt{2}} R_o H^2 \sqrt{\sigma_o \rho} \right) \frac{a^2 / R_o^2}{J(a/R_o)} \quad (A7)$$

using  $\tau^2 = R_o^2 \rho H / 2M_o = 2R_o^2 \rho / \sigma_o$ , from [3,4]. Thus from Eqs. (A4, A5, A7) we obtain final deflection and response time as function of  $I$ , in physical terms for comparison with test results.

To find the effect of changes in  $a$ , which is somewhat uncertain, alternative values are inserted in Eq. (A7). In the same way one may determine the effects of changes in  $\sigma_o$ ,  $\dot{\epsilon}_o$ , or  $R_o$  by making changes merely in Eq. (A7). This is possible despite the fact that such changes would alter the parameter  $\alpha$  and, in principle, require a recalculation of the response functions  $F_1$ ,  $F_2$  of Eqs. (A4). The parameter  $\alpha$  appears in these equations as  $\alpha^{1/n'}$ , where  $n' = \nu n$  and the factor  $\nu$  generally exceeds 2, depending on the strain rate. Since  $n$  generally exceeds 5, it is seen that the dependence of the mode response on  $\alpha$  is exceedingly weak. (For example, a 50 percent change in  $\alpha$  means a 4 percent change in  $\alpha^{1/10}$ ). Hence a "master" response curve for representative values of the material parameters and plate radius will serve for considerable ranges of these quantities. This enhances the computational efficiency of the mode technique.



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Tests are described on circular plates of mild steel and commercially pure titanium loaded impulsively by means of explosive sheet. Three loading geometries were used, with magnitudes such that final deflections in the range from one to about seven plate thicknesses were produced. Clamping against radial as well as transverse deflections at the edge was provided. Both materials exhibit strong plastic rate sensitivity. Parameters describing this behavior were obtained from stress-strain tests at low to intermediate rates together with published data for high strain rates. The measured final deflections and response times are compared with predictions of the mode approximation technique as extended to large deflections of viscoplastic structures.

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	tests						
	approximation techniques						